

## Mth 445 Real Analysis Review Guide for 2<sup>nd</sup> Test Test Date Thursday March 12th

### Section 14: Compact Sets

Definition of open cover, finite open cover, compact set.

Show a given set  $S$  is NOT compact.

State Heine-Borel Theorem. State Bolzano Weierstrass Theorem.

### Section 16 Convergence

Definition of a sequence.

Definition of convergence of a sequence -- state formally and then illustrate graphically.

Using the definition for convergence, show that a given sequence converges to a specific limit.

### Section 17 Limit Theorems

Know properties of limits stated in Theorem 17.1 -- be able to complete the proof of one of the parts. Use the Theorem to determine the limit of a sequence (identifying which parts of the theorem you used.)

### Section 18 Monotone Sequences and Cauchy Sequences

Define increasing, decreasing, monotone sequence.

State the Monotone Convergence Theorem.

Show using induction that a recursively defined sequence is monotone and bounded (like Example 18.4 and exercises).

Definition of Cauchy sequence.

\*Statement of Theorem 18.12 – Cauchy Convergence Criterion. Fill in the blanks for an outline of proof of the Cauchy Convergence Criterion:

### Section 19 Subsequences

Statements of Theorems 19.4 and 19.7 on convergent subsequences.

Definition of subsequential limits,  $\limsup$  and  $\liminf$  of a sequence.

For a given sequence answer the following: (From ideas in section 16, 18, 19)

- Is this sequence bounded (justify)?
- Is this sequence monotone (justify)?
- Is it Cauchy (justify)?
- Is it convergent (justify)?
- Give the set of subsequential limits.
- What is the  $\limsup s_n$  ?
- What is the  $\liminf s_n$  ?

## Chapter 5

### Section 20 Limits of Functions

Definition 20.1 of limit of function.

Apply the definition to problem like examples 20.5, 20.6 and exercise 20.4

Theorem 20.8: Relationship of limit of a function to sequential limits.

Apply the theorem to show that a limit of a function at a point does not exist – like example 20.11

Apply Theorem 20.13 Limit formulas to determine limits of problems like Practice 20.15, Example 20.16 and Exercise 20.3

### Section 21 Continuous Functions

Definition 21.1 State and apply to problems like Exercise 21.4

Statement of Theorem 21.1

Dirichlet and modified Dirichlet Function (Examples 21.8 and 21.9)

Statement of Theorem 21.1 and Theorem 21.12