Limit Theorems

- **Theorem 17.1**: Suppose (s_n) converges to s and (t_n) converges to t. Then

  a) $(s_n + t_n)$ converges to $s + t$
  b) (ks_n) converges to $k\,s$
  $(s_n + k)$ converges to $s + k$
  c) $(s_t\, t_n)$ converges to $s\,t$
  d) (s_n / t_n) converges to s / t

  Proof of a) Relies on the triangle inequality.

  Choose N_1 such that $|s_n - s| < \varepsilon/2$ for all $n >= N_1$.
  Choose N_2 such that $|t_n - t| < \varepsilon/2$ for all $n >= N_2$.

  Then by Triangle inequality, for $N = Max(N_1,N_2)$

  $|(s_n + t_n) - (s + t)| <= |s_n - s| + |t_n - t| < \varepsilon/2 + \varepsilon/2 = \varepsilon$ for all $n >= N$.

  The proof of c) relies on the Triangle Inequality with a little more finesse and the fact (theorem) that every convergent sequence is bounded.

  Part d) is proven from part c) after showing that $\left(\frac{1}{t_n}\right)$ converges to $\frac{1}{t}$

Practice 17.3: Show each step in application of 17.1

Definition 17.9:

A sequence (s_n) **diverges to +∞** if for every real M there exists an N such that s_n > M, for all n >= N.

A sequence (s_n) **diverges to -∞** if for every real M there exists an N such that s_n < M, for all n >= N.