Bolzano-Weirstrass Theorem

If a bounded subset S of R contains infinitely many points, then S has at least one accumulation point.

Proof:

(By contradiction)

Assume S is a bounded subset containing infinitely many points.

Suppose S has no accumulation points.

Then S must be closed (why?)

By ___________________________, S is compact.

Since S has no accumulation points, any *x* in the set S is not an accumulation point.

Hence *x* has a neighborhood N(*x*) about it that has only *x* in common with S. (why?)

Construct such a neighborhood for each *x* in S.

This collection of neighborhoods forms an open covering, hence admits a finite subcovering, say $N(x_1)$, $N(x_2)$, …, $N(x_n)$. (Why)

Thus $S = S \cap (N(x_1) \cup N(x_x) \cdots \cup N(x_n)) = \{x_1, x_2, \ldots x_n\}$

This is a contradiction of S being infinite.