Braketing Methods for Finding Roots
Chapter 2(Section 2.2)

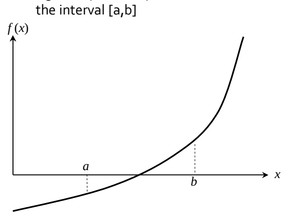
Definition

Let *f* be a real-valued function.
Any number *r* for which *f*(*r*) = 0 is called a *root* of the equation f(*r*) = 0 and a *zero* of the function *f*.

By the **Intermediate Value Theorem**, we know that if f is a continuous function on an interval [*a*,*b*], and *f*(*a*) and *f*(*b*) have opposite signs, then there will be a value *r* in the interval [*a*,*b*] where *f*(*r*) = 0.

Example

This function is real, continuous and f(a) and f(b) have opposite signs (Equivalently f(a)f(b) <0). Thus f has at least one zero in the interval [a,b]



Bisection Algorithm

Find a,b such that f(a)f(b) < 0.
Repeat the following
Let c = (a+b)/2 (midpoint of interval [a,b])
If f(c) is approximately 0 stop – c is our root.
Otherwise
If f(b)f(c) >0 (zero is in the left subinterval)
let b = c.
If *f(b)f(c)* < *0* (zero is in the right subinterval)
let a = c.
Until the subinterval [a,b] is small enough

Let c = (a+b)/2 – this is the approximation to zero of f (i.e. f(c) = 0)

Advantages and Disadvantages of the Bisection Method

Advantages
- Always convergences to a root (since the root is always in the subintervals generated).
- We can measure the error in our solution: If r is the root and $\{c_n\}$ is the sequence generated by the method, then

$$|r - c_n| \le \frac{b-a}{2^{n+1}}$$

Disadvantage
- Slow convergence

Improvement: False Position Method

Instead of choosing the midpoint of the interval to test, choose the point cn where the secant line through (a,f(a)) and (b,f(b)) intersects the *x* axis.

$$c = b - \frac{f(b)(b-a)}{f(b) - f(a)}$$

This method has the potential to converge more rapidly (as we will see apply in the lab).