M439 Numerical Analysis Lab: Euler's and Heun's Methods for Numerical Solution of First Order Ordinary Differential Equations

m-files for this lab: euler.m heun.m

Comparing Heun's and Euler's Method

1. Consider the initial value problem we worked on in class.
   y' =3t+3y
   y(0) = 1

An analytic solution to this is y(t) = $\frac{4}{3}e^{3t} - t - \frac{1}{3}$.

2. Approximate the solution to this problem on the interval [0,1] using euler.m on the interval 0 1 with a step size of .2
and then .1 as follows:

First create the m-file function (save as f1.m) which contains the code:

```
function z = f(t,y)
   z = 3*y+3*t;
```

3. Call the euler function first with
   E1 = euler(@f1,0,1,1,5)

4. Then call with
   E2 = euler(@f1,0,1,1,10)

5. Verify that you got the same answers as we did in class for the first steps (we only went to t = .4 in class, not all the way to t = 1)

6. Next run Heun's algorithm on the same subintervals

   H1 = heun(@f1,0,1,1,5)
   H2 = heun(@f1,0,1,1,5)

7. Repeat the above for

Create an m-file called f3.m that contains the right hand side of your differential equation for exercise 3 in Problem Set 13:

```
function z = f(t,y)
  z = exp(-2*t) - 2*y;
```

Run Euler's method on this differential equation
E1 = euler(@f3,0,1,1/10,5)
Then call with
E2 = euler(@f3,0,1,1/10,10)

8. Next run Heun's algorithm on the same subintervals

   H1 = heun(@f3,0,1,1/10,5)
   H2 = heun('@f3,0,1,1/10,10)

9. Plot the exact solution and your approximations with 10 subintervals on the same graph for comparison for both Euler and Heun:

```
ezplot('.1*exp(-2*x)+x*exp(-2*x)',0,1)
hold on
plot(E2(:,1),E2(:,2),'g-')
plot(H2(:,1),H2(:,2),'r-')
```

11. Clearly Heun's method is much more accurate.
Why does the Euler error increase as we increase our steps for awhile, and then decrease again?

Exercises to Turn in For Grade Worth 25 points. Think Before You Leap.

Mathematical Model for Epidemics: Assume there is a community of N persons that contains P infected individuals and N-P uninfected individuals. Since there are P*(N-Q) possible contacts between the groups giving opportunity to transmit the disease. If we let y(t) denote the number of infected individuals at time t, and y(0)= y_0 , then the rate of spread of the disease $\frac{dy}{dt}$ is proportional to the number possible contacts at time t between infected and uninfected and so is modeled by the differential equation with initial value:

$$\frac{dy}{dt} = ky(N - y), \quad y(0) = y_0$$

Assume that N is 1500 and k = 0.0003, y(0) = 25.

Put your commands in an m-file called **epidemic.m** that contains all of the commands to accomplish the following: (Document with a comment (%) which part doing.)

a) Use Heun's method with an increment size of h = .2 to approximate the solution over [0,60]. You will need to create an m file (call it **fa.m**) to define the right hand side of the differential equation with these values for k and N.

b) Plot the graph of the approximate solution in part a.

c) Estimate the average number of individuals infected by using the **mean** function in MATLAB to average the y values generated by Heun

d) Estimate the average number of individuals infected a second way:

1. Fit a logistic curve to the data points generated (using the method of Least Squares to fit a logistic curve $y = \frac{N}{1 + Ce^{At}}$ to these data points. Plot this graph (should look like the other).

2. Apply the Mean Value Theorem for integrals to find the average of this function over the interval [0,60] -- use either your simpsum file or MATLAB's quad function to do the numerical integration). You will need to define an m-file fd.m to contain the function that you found to fit the data points).

e) Compare your two answers for the average number of infected individuals
email to me the m-files:

epidemic.m, fa.m, fd.m and in the body of your message give the answers you found for a, c, and d part 1 (the curve) and d part 2 (mean).