

**Numerical Analysis Problem Set 13 Euler's Method and Heun's Method**  
**Show each step in these problems.**

1. Apply Euler's method to the following initial value problem

$$\frac{dy}{dt} = t^2 - y$$
$$y(0) = 1$$

- Do two steps with  $h = 0.2$
- Do four steps with  $h = 0.1$
- Compare (compute the error) in the estimate you get for  $y(0.4)$  with the exact solution to the above:

$$y(t) = -e^{-t} + t^2 - 2t + 2$$

2. Apply Heun's method to the same initial value problem in exercise 1 and with the same step sizes  $h$ . Also compare to the exact (compute your error).

3. Apply Euler's method to the following initial value problem

$$\frac{dy}{dt} = e^{-2t} - 2y$$
$$y(0) = 0.1$$

- Do two steps with  $h = 0.2$
- Do four steps with  $h = 0.1$
- Compare (compute the error) in the estimate you get for  $y(0.4)$  with the exact solution to the above:

$$y(t) = \frac{e^{-2t}}{10} + te^{-2t}$$

4. Apply Heun's method to the same initial value problem in exercise 3 and with the same step sizes  $h$ . Also compare to the exact (compute your error).