Numerical Analysis Problem Set 12 Introduction to Differential Equations (reference Class Notes)

1. a) Show by substitution that the following function

$$y(t) = Ce^{-t} - t - \frac{1}{3}$$

is a solution to the differential equation

$$\frac{dy}{dt} = 3y + 3t .$$

b) Determine the value of C for the initial condition y(0)=1.

2. a) Show by substitution that the following function

$$y(t) = Ce^{-2t} + te^{-2t}$$

is a solution to the differential equation

$$\frac{dy}{dt} = e^{-2t} - 2y .$$

b) Determine the value of C for the initial condition y(0)=2.

3. Find a solution to the following separable initial value problem:

$$\frac{dy}{dt} = 3t^2 + \sin(t)$$
$$y(0) = 2$$

4. Find a solution to the following separable initial value problem:

$$\frac{dy}{dt} = \frac{1}{1+t^2}$$
$$y(0) = 0$$

5. Radioactive decay.

a) Consider the initial value problem for exponential decay. Exponential decay occurs when the rate of change is proportional to the amount of material at time t. The initial value problem governing exponential decay is given by

$$\frac{dy}{dt} = ky$$, where k is a negative constant.

$$y(0) = y_0.$$

Solve this initial value problem.

b) The half-life of Carbon-14 is 5730 years. Find the k in your answer to a) that models carbon-14.

c) Find the age of a bone that has only 70 percent (.70) of the carbon-14 present when it was alive.

—