Numerical Analysis Semester 081 Problem Set 11

Reference: For problem 1: 2.3 Differentiation of Discrete Functions

For problems 2 – 4 Sections 7.2 Trapezoidal Rule and 7.3 Simpson's Rule

1. a) Write down the equation for the second degree Newton polynomial interpolating the points x_0, x_1, and x_2.

b) Differentiate this to get a formula for the first derivative of the Newton Polynomial which can be used to approximate f '(x)

c) Let $x_0 = x$, $x_1 = x + h$, and $x_2 = x + 2h$ to derive the forward difference formula of order 2 for approximating f '(x) with points (x, f(x)), (x + h, f(x + h)), and (x + 2h, f(x + 2h)).

2. Use the Multi-Segment (Composite) Trapezoidal Rule to approximate the following integral -- show all work clearly. Use a subinterval size h = .25 (hence five data points)

$$\int_0^1 \sin(x^2)dx$$

3. Use the Multi-Segment (Composite) Simpson Rule to approximate the same integral as in 2 -- using the same subinterval size (hence five data points).

4. Determine the minimum number of data points and interval width needed for to guarantee an accuracy of 10^{-8} in the computation of

an approximation to $\int_0^2 xe^{-x}dx$

a) Using the Multi-Segment (Composite) Trapezoidal Rule.
b) Using the Multi-Segment (Composite) Simpson Rule.