M439 Numerical Analysis Lab: Numerical Differentiation and Error Analysis for Optimum Step Size

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1. Save these files to your N:\m439 directory : difflim.m   f.m

2. Make N:\m439 your working directory in MATLAB

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3. Forward Difference Quotient: Error Analysis and Optimum Step Size

When computing an approximation to the derivative, the error includes both truncation error and computer round-off error. For example, iIn the forward difference quotient formula, since we have a certain round-off error in the computation of the function of f(x+h) - f(x) -- this error is “magnified” when we divide by a small value of h. Eventually when h is small enough the round-off error introduced exceeds the benefits of the reduction in the truncation error so its time to quit – an optimal value for h has been found.

Use the ***forward difference quotient*** formula to approximate the derivative of exp(*x*) at x =1. Since MATLAB uses more than 9 digits of accuracy in its computations it should be able to use small values of *h* before loss of significant digits caused by computer round-off error kicks in. Find the optimal value of h and the best corresponding approximation.
Set format to long and then enter approximations:

```
format long
h = 10^-1, d = (exp(1+h) - exp(1))/h,  d - exp(1)
h = 10^-2, d = (exp(1+h) - exp(1))/h,  d - exp(1)
etc.
```

Stop when h is small enough that error **d - exp(1)** is increasing.

Optimal value for h = __________;   approximation to derivative for this h:
____

Error: ____________________________

2. Central-Difference Formula of order O(h^2).

Repeat the above for the Central-difference Formula

```
h = 10^-1, d = (exp(1+h) - exp(1-h))/(2*h),  d - exp(1)
```

Optimal value for h = ___________; approximation to derivative for this h: _________________ Error: _____________________________

How do the two methods compare in terms of rate of convergence and error?

____ ______

____ ______

3. The m-file function **difflim**

This file implements the Central Difference Formula of order O(h^2).
To call this function, we must first have an m-file containing the function we want to differentiate.
The file **f.m** contains the function cos(x).

Execute the following sample call:

```
[L n ] = difflim('f',.8,10^-7)
```

This numerically differentiates the function stored in the file f.m, at the point 0.8, with a tolerance of error = 10^-7. L stores the tabulation of the results of the differentiation, and *n* is the number of iterations. The first column of L stores the "*h*" values, the second column stores the numerical approximation to the derivative, and the last column stores the approximate error (computed as the difference of the last two approximations to the derivative).

Check the accuracy of the approximation by calculating what we know to be the derivative of cos(x) at .8:

```
-sin(.8)
```

-sin(.8) ______________ value returned by difflim _____________

difference _________________

4. Central Difference Formula of Order $O(h^4)$

This gives us a faster converging formula for the numerical differentiation. (However it requires the use of four approximating points for the function f)

Modify the algorithm in file difflim.m to implement this formula -- *save the new m-file as* **difflim2.m**
(Just change the two lines **`D(n+1)=(feval(f,x+h)-feval(f,x-h))/(2*h);`** and
`D(n+2)=(feval(f,x+h)-feval(f,x-h))/(2*h);` to reflect the new formula.)

Test your program on the same input as before -- Do you get faster convergence? _________, which is expected since

____
_________________.

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5. Consider the following problem

The theoretical model for the current for a particular electrical circuit is given by $I(t) = 10e^{-t/10}\sin(2t)$. The derivative dI/dt is needed for certain formulas for circuits such as computing voltage. In this exercise we first approximate the derivative of I(t) at 1.2 based on the numerical data in the following table.

t	**I(*t*)**
1.0	8.2277
1.1	7.2428
1.2	5.9908
1.3	4.5260
1.4	2.9122

A) First Method: Use Central-difference Formula of order $O(h^2)$. Here we are using h = .1
From the table we need to compute:

df = (4.5260-7.2428)/(2*.1)

Value for I'(t) at 1.2: _________________

B) Second Method: Repeat the calculation this time using the Central-difference Formula of order $O(h^4)$

```
df = (-2.9122 + 8*4.5260 -8*7.2428 + 8.2277)/(12*.1)
```

Approximation for I'(t) at 1.2: ________________

C) Compare your answer with the theoretical value $I(t) = 10e^{-t/10}\sin(2t)$. To do this, we need to differentiate symbolically I(t), evaluate the result at 1.2, and then use the formula for E(t).

```
syms t
sdf = diff(10*exp(-t/10)*sin(2*t),t)
dfval = subs(sdf,t,1.2)
```

You should get:

```
dfval =
 -13.67927322348736
```

```
Ival = subs(10*exp(-t/10)*sin(2*t),t,1.2)
```

I ' (1.2) _________________________