

M439 Lab on Quadrature (Numerical Integration) in MATLAB

The Composite Trapezoidal Rule

1. Recall the Composite Trapezoidal Rule

$$\int_a^b f(x) dx \approx \frac{h}{2} [f(a) + f(b)] + h[f(x_1) + f(x_2) + \dots + f(x_{n-1})]$$

Exercise: Write a MATLAB m-file `trapsum.m` to carry out the Composite Trapezoid Rule.

The heading will be: `function quad = trapsum (f, a, b, n)`

Here **a** and **b** are the endpoints of the interval of integration, **f** is the function to be integrated, and **n** is the number of subintervals (hence n+1 data points)

An outline for the body of the function is:

Set **h** to the size of the subintervals

* Set up the vector **xvector** with the values of the endpoints of the subintervals

Set **fvector** as the vector containing the function **f** evaluated at data points in **xvector**

** Assign to the variable **quad** the value of approximation to the integral using the composite trapezoid formula.

Hints:

* use the function `linspace` (enter `help linspace` to see how it works).

** take advantage of the `sum` function

2. Test your Trapezoidal method by using it to estimate the integral of $5x^4$ over $[0,1]$ (which we know has the true answer 1), using the given values of n , and record the error in your approximation. To do this you will have to create a file called "f.m" (or any other name you might want) that contains the code to create the function $y = 5x^4$. You will then call your function as above with

```
trapsum (@f,0,1,3)
```

number of subintervals	h (size of subinterval)	Trapezoid Approx.	Error
2	.5		
10	.1		
1001	.01		
1000	.001		

3. The error formula for the Trapezoidal Rule is given by $\frac{-(b-a)f^{(2)}(c)h^2}{12}$, where $a < b < c$ which is clearly $O(h^2)$

Are your errors consistent with this formula? (Justify)

Composite Simpson's Rule

4. Exercise: Create a new m-file called **simpsum.m** that implements the Composite Simpson Rule

$$\int_a^b f(x) dx \approx \frac{h}{3} (f(a) + f(b) + 4[f(x_1) + f(x_3) + \dots + f(x_{n-2})] + 2[f(x_2) + f(x_4) + \dots + f(x_{n-3})])$$

This is a little more complicated -- we have to set up a vector **xodd** storing the x-values of the "inner odd data points" and then evaluate our function f at these points. Similarly we have to set up a vector **xeven** storing the x-values of the "inner even data points" and then evaluate our function f at these points.

`function quad = simpsum(f,a,b,n)`

`% simpsum(@f,a,b,n) -- f is the function to be integrated`
`% over the interval [a,b] using n as the number of subintervals, which must be even`
`% (Thus there are n + 1 data points)%`

`%-----`

`%Set up the width of the subintervals`
`h = (b - a) / n`

`%set up the xodd data points`
`xodd = linspace(a+h,b-h,(n-1)/2)`

`%set up fodd -- evaluate f at these data points`
`fodd = feval(f,xodd)`

`%set up the xeven data points`
`xeven = linspace(a+2*h,b-2*h,(n-3)/2)`

`%set up feven -- evaluate f at these data points`
`feven = f(xeven)`

`%approximate using the Simpsons's composite rule formula:`
`quad = (h/3)*(f(a) + f(b) + 4*fodd + 2*feven);`

5. Test your m-file on the function $5x^4$ on the interval $[0,1]$.

n (number of subintervals – even)	h (size of interval)	Simpson Approx.	Error
2	.5		
10	.1		
100	.01		
1000	.001		

The error for the Composite Simpson's Rule is $O(h^4)$. Are your Errors consistent with this? Explain

6. Compute the answer to the following problem:

Find the surface area of the solid of revolution obtained by rotation the curve $y = e^{-x}$ about the x-axis for $0 \leq x \leq 1$. Recall the formula for the surface area of the solid of revolution for $y = f(x)$,

$$a \leq x \leq b \text{ rotated about the x-axis is } area = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} dx$$

First set up an m-file **f.m** that contains the function $y = \exp(-x) \cdot \sqrt{1 + \exp(-2 \cdot x)}$. Then use both of your functions trapsum and simpsum with 100 intervals.

e.g. `2*pi*trapsum(...)`

Answer with trapsum _____ Answer with simpsum _____

7. Compare your answer against answer with built-in MATLAB function quad with a tolerance of $0.5 \cdot 10^{-10}$.

`2*pi*quad(@f3,0,1,.5*10^-10)`

Answer with quad: _____