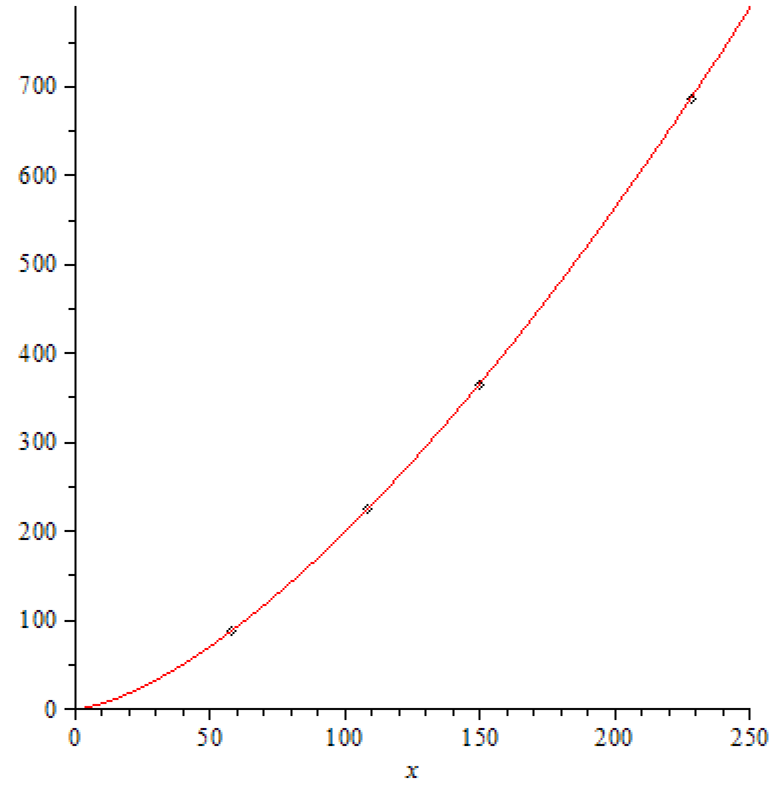
Chapter 5

Curve Fitting

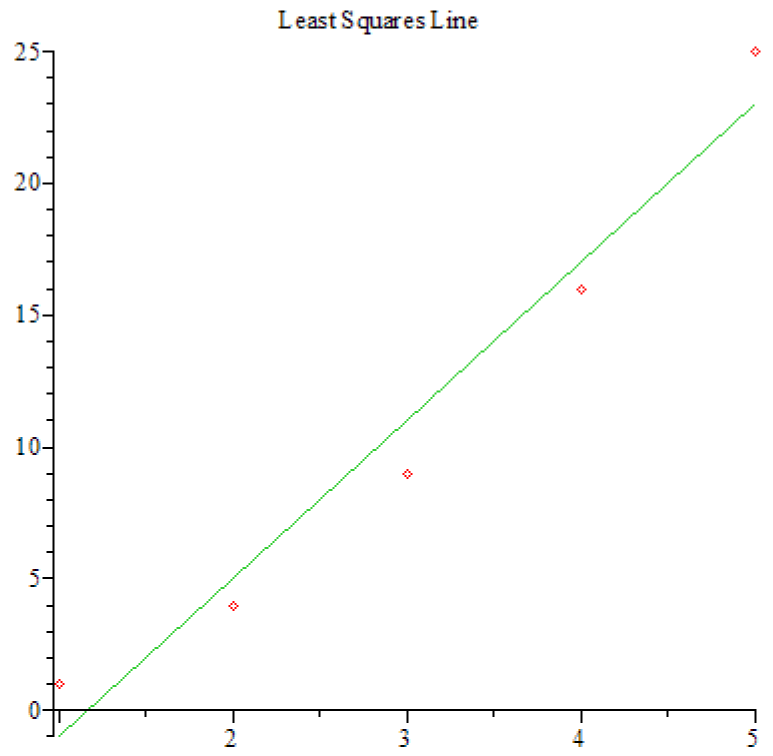
Least Squares Fit Example

Kepler's 3rd Law for planetary motion: Data points for (Distance from sun, Orbital periods) for Mercury, Venus, Earth, and Mars fitted with curve:

$$T = 0.199769x^{3/2}$$



Section 5.1 Least Squares Line



Least Squares Method

The least squares method minimizes the sum of the square of the errors (also called residuals or deviations) in the model:

$$E(A,B)=\sum_{k=1}^{n} d_k^{\,2}=\sum_{k=1}^{n}\left(Ax_k+B-y_k\right)^2$$

This produces a unique line:

$$y=Ax+B$$

Theorem 5.1 The Normal Equations

The A and B for the least squares line $y = Ax+B$ fitting the data points (x_1,y_k), for $k=1,..,N$ is found by solving the Normal Equations:

$$(\sum_{k=1}^{N} x_k^2)A + (\sum_{k=1}^{N} x_k)B = \sum_{k=1}^{N} x_k y_k$$

$$(\sum_{k=1}^{N} x_k)A + NB = \sum_{k=1}^{N} y_k$$

Theorem 5.2 The Power Fit $y = Ax^M$

The coefficient A for the least-squares power curve $y = Ax^M$, for given M, is given by

$$A = \frac{\sum_{k=1}^{N} x_k^M y_k}{\sum_{k=1}^{N} x_k^{2M}}$$

Data Linearization: Exponential Model

Given the data points (x_1, y_k),

for k=1,..,N, we want to fit the exponential curve:

$$y = Ce^{Ax}$$

Taking the natural log of both sides of this equation gives

$$\ln(y) = Ax + \ln(C)$$

We can then fit the data points $(x_1, \ln(y_k))$ with the least squares line with constants A and B = ln(C).

In the above equation, $C = e^B$.

Change of Variables Method (Data Linearization)

- Data linearization can be applied to other equation
- Example -- The logistic curve: $y = \frac{L}{1+Ce^{Ax}}$
- Rewrite as $\frac{L}{y} - 1 = Ce^{Ax}$
- Take natural log of both sides gives the linearization:

$$\ln\left(\frac{L}{y} - 1\right) = Ax + \ln(C)$$

- This means we must fit the data points (x_k, $\ln(L/y^k - 1)$) with a least squares line Y=AX+B.
- The C in the logistic equation will be e^B. (The A will be the one found in the fit.)