**MTH 341 Practice Questions for Third Exam Semester 091**

1. Suppose that X is a continuous random variable whose density is

$$f(x) = \begin{cases} 4x^3, 0 \le x \le 1 \\ 0, \text{otherwise} \end{cases}$$

Find

a) E(X)

b) Var(X)

c) The distribution function F($x$) for X.

d) The distribution function F(y) for Y = $e^X$, where X is as above.

2. Suppose X is a random variable that is **uniformly** distributed on the interval [1,3]

a) Calculate the probability P[1 < X < 2]

b) Write down the density function for X

c) Write down the distribution function for X

3. The density function for the standard normal ( mean =0, standard deviation = 1) random variable is given by

$$a)\ \frac{1}{\sqrt{2\pi}} e^{-\frac{x}{2}} \quad b)\ \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad c)\ \frac{1}{\sqrt{2\pi}} e^{\frac{x^2}{2}} \quad d)\ \frac{1}{\sqrt{2\pi}} e^{\frac{x}{2}} \quad e)\ \text{none of these}$$

4. If X is a Normal random variable with mean $\mu$ and variance $\sigma^2$, then standardized random variable X* = _______________

5. Use the normal approximation to the binomial distribution (with continuity correction) to solve the following problem: Suppose that 80% of the patients with a certain disease can be cured with a certain drug. What is the approximate probability, that of 100 such patients, at least 85 patients will be cured.

6. Answer for the following properties of a Poisson process with rate $\lambda$.

a) T F Events of occurrences over disjoint intervals are independent.

b) The average rate of arrivals per unit of time is equal to __________.

c) The **number of arrivals** in time $t$ has a(n) _________________ distribution with parameter ______.

d) P(an occurrence in a sufficiently small interval $\Delta t$) is approximately equal to ______

e) The waiting times $W_1$, $W_2$, … between occurrences are each independent with a(n)____________________ distribution.

7. Suppose that telephone calls arrive at a station at the rate of 30 per hour (i.e. 0.5 per minute) according to a Poisson process.
Find:
a) The probability that the time between the fifth and sixth telephone call is at least 3 minutes.
b) The probability that the third telephone call arrives ***after*** time 4 minutes.
c) The expected time at which the first telephone call arrives.

8. Which of the following is the correct statement of the memoryless property for the exponential random variable X:
a) $P(X > t) = e^{-\lambda t}$
b) $P(X > t, X > s) = e^{-\lambda ts}$
c) $P(X > t + s \mid X > s) = P(X > t)$
d) $P(X > t, X > s) = \min(P(X > t), P(X > s))$

9. Prove the memoryless property above.

10. A fair die is tossed three times. Let X be the number of sixes on tosses 1 and 2. Let Y bet the number of sixes on tosses 2 and 3. ( For example with outcome 6 6 3, X = 2 and Y = 1. )
a) Write down the in table form, the **joint distribution** of X and Y; including also the **marginal distributions** for X and Y as well.
b) What is the probability that X = 0?
What is the probability that Y = 0?
What is the P[X = 0, Y = 0]?
c) What does part b) say about the independence of X and Y? (Explain by using the definition of independence of random variables)

11. Two people are to meet at the center of a mall between 10:00 a.m. and 10:15 a.m. If they each arrive at random times within this period what is the probability that they will meet within 5 minutes? Explain your work, using a sketch to support your computations.

12. Let the joint density of X and Y be given by

$$f(x,y) = \begin{cases} 6e^{-(2x+3y)}, & \text{x > 0, y > 0} \\ 0, & \text{otherwise} \end{cases}$$

a) Calculate P( X < Y)
b) Calculate the marginal density of $f$ with respect to X, $f_X(x)$
( Note that $e^{-(2x+3y)} = e^{-2x} e^{-3y}$ )
c) Calculate the marginal density of $f$ with respect to Y, $f_Y(y)$
d) Are X and Y independent random variables? ( Yes, No ) Justify your answer: