MTH 341 Probability Sample Questions for Second Exam Semester 091

1. Which of the following is the correct definition the distribution function F of a random variable X:

a) $F(t) = P(t^+) - P(t^-)$
b) $F(t) = P(X \le t)$
c) $F(t) = P(X < t)$
d) $F(t) = P(X = t)$

2. Which of the following **may** not be ***true*** for the distribution function F(X):

a) $\lim_{x\to\infty} F(x) = 1$.
b) $\lim_{x\to-\infty} F(x) = 0$
c) F is nondecreasing.
d) F is left continuous
e) F is right continuous.

3. For a discrete random variable X, its probability mass function p(x) is defined by

a) $p(x) = \sum_{i<=x} xP(X = i)$
b) $p(x) = P(X = x)$
c) $p(x) = P(X \le x)$
d) $p(x) = \sum_{i<=x} P(X = i)$

4 Consider the distribution function of a random variable X defined by:

$$F(x) = \begin{cases} 0, & x < 0 \\ x/4, & 0 \le x < 2 \\ 3/4 & 2 \le x < 3 \\ 1, & 3 \le x \end{cases}$$

Calculate the following probabilities:

P(X = 1) P(X=2) P(X ≤ 2.5) P(X < 3) P(1 ≤ X ≤ 2)

5. Toss a die twice. Let X be the number of sixes rolled.

a) Write down the probability mass function $p(x)$ for X:

x			
$p(x)$			

b) Sketch the graph of the **distribution function** F(x):

c) Compute E(X), the expected value of X.

d) Compute Var(X), the variance of X.

6. Suppose X is a random variable with E(X) = 6 and Var(X) = 2. Calculate the following:

a) Var(2X - 3)

b) E(2X -3)

c) $E(X^2)$

d) σ_X (standard deviation of X)

7. Explain each of the following properties for the Poisson process with parameter λ:

a) The property of stationarity.

b) The property of independent increments.

c) The property of orderliness.

d) The meaning of the parameter λ

8. Suppose that earthquakes happen at the rate of 2 per month according to a Poisson process. Find:

a) The probability that exactly 5 earthquakes happen during 3 months.

b) The probability that at least 2 earthquakes happen during 4 months.

c) Given that 4 earthquakes happen during the previous 2 months, what is the probability at least 2 earthquakes happen during the next 4 months.

9. a) Suppose that 2% of the families in a very large city are HIV positive. A sample of 60 random persons are tested for HIV. Using the Poisson approximation to the binomial distribution , compute the approximate probability that 2 or less people in the sample are shown to be HIV positive?

b) Explain why it is appropriate to use the Poisson approximation to the binomial distribution in this case.

10. Suppose we have a pack with 10 dark chocolate Hershey kisses and 30 white chocolate Hershey kisses. You repeatedly pick a Hershey kiss at random from the pack (using great will power to ***replace after each pick***).

a) Let X = the number of picks required to get your **first** dark chocolate Hershey kiss. Then X has the

i) Binomial distribution
ii) Hypergeometric distribution
iii) Poisson distribution
iv) Geometric distribution
v) Negative Binomial distribution
vi) Dark Chocolate distribution

b) Calculate P(X = 5)

c) What is the average number of picks required to get the first dark chocolate kiss in the above model?

d) Let Y = the number of picks required to finally get **3 dark chocolate kisses**
Y has the

i) Binomial distribution.
ii) Hypergeometric distribution.
iii) Poisson distribution.
iv) Geometric distribution.
v) Negative Binomial distribution.
vi) Dark Chocolate distribution

e) Calculate P(Y = 6).

f) Calculate E(Y)

g) Let Z = the number of dark chocolates kisses picked in 6 tries. Then Z has the

i) Binomial distribution.
ii) Hypergeometric distribution.
iii) Poisson distribution.
iv) Geometric distribution.
v) Negative Binomial distribution.
vi) Dark Chocolate distribution

h) Calculate P(Z=2)

i) Calculate E(Z)

11. Suppose in we again have a package with 10 dark chocolate kisses and 30 white chocolate kisses. You randomly pick 6 pieces of candy (***without replacement this time)***. Let X be the number of dark chocolate kisses in our sample of size 6.

a) Then X has the

i) Binomial distribution
ii) Hypergeometric distribution
iii) Poisson distribution
iv) Geometric distribution
v) Negative Binomial distribution
vi) Dark chocolate distribution

b) P(X = 3) = _____

12. a) Give the formula for the following in terms of the ***density*** function $f(x)$ for a continuous random variable X:

$P(a \le X \le b)$ = _____

b) Draw a sketch of what this means for a given density function $f(x)$ and an interval (a,b). (i.e. Shade area that represents $P(a \le X \le b)$)

13. Given a distribution function $F(x) = 1 - e^{-5x}$, $x \ge 0$,
calculate its density function $f(x)$ (Show work)

14. Suppose that X is a continuous random variable whose probability density function is

$$f(x) = \begin{cases} cx^2, 0 \le x \le 1 \\ 0, \text{otherwise} \end{cases}$$

Find

a) c b) P[0 < X < 1/2]