MTH 244 Linear Algebra Review Guide First Test Semester 112

1. 1 Matrices and Systems of Linear Equations
Geometrical interpretation of system of equations in two variables and their solutions
-- when unique solution, infinite number of solutions, or no solution.
Recognize a linear equation, a linear system of equations, and solution or solution set to a linear system of equations.
Know when a system is consistent or inconsistent.
Write augmented matrix, matrix of coefficients, and matrix from (section 2.2) for a linear system
3 elementary operations on a linear system and the 3 corresponding elementary row operations on a matrix

1.2 Gauss-Jordan Elimination
Reduced row echelon form for a matrix -- recognize.
Method of Gauss-Jordan elimination for solving a system of equations
-- apply to find general solution, unique solution, or show no solution
leading variables and free variables (non-leading), pivots
Homogeneous Systems of Linear Equations
Trivial solution to homogeneous system.
Theorem 1.1, 1.2

Section 2.1 Addition, Scalar Mult., and Mult. Of Matrices thru page 42
Entry or element of a matrix -- double subscript notation
Size of a matrix
Square matrix, main diagonal of matrix
Equality of matrices.
Matrix operations: addition, subtraction, multiplication by scalar, matrix multiplication,
Negative of matrix, zero matrix.
Identity matrix, Diagonal matrix

Section 2.2 Properties of Matrix Operations
Properties of matrix addition, scalar multiplication, multiplication -- recognize and apply.
Determine the size of the result and number of multiplications required to perform a chain of matrix
Power of Matrix
Matrix form for a system of linear equations: AX=B

Section 2.3 Symmetric Matrices through page 62
Transpose of a matrix, definition and properties
Symmetric matrix
Definition and properties of trace of a square matrix

Section 2.4 -- through page 74
Inverse of a matrix -- definition and notation, invertible matrix.
Be able to prove Theorem 2.7 (uniqueness of inverse)
Use Gauss-Jordan elimination to find inverse of a matrix or show it doesn't exist (Examples 2,3)
Statement and proof of Theorem 2.8
Apply Theorem 2.8 to find solution to a system AX=B given inverse of A and B
Properties of matrix inverses (page 72) -- know and be able to apply to problems. Be able to prove property 3.

Section 3.1 Introduction to Determinants
Terms: ijth Minor, ijth Cofactor.
Know how to calculate a determinant in terms of any row or column cofactor expansion -- by hand.

Practice Questions – Note this does not take the place of studying above topics !!!

Sections 1.1 and 1.2

T F 1. It is possible for a linear system of equations to have exactly 3 solutions.
T F 2. A homogeneous system will always have at least the trivial solution.

3. Which of the following are linear equations (circle any):

$\cos(\pi)x + y - z = 6$ $\qquad$ $4x^2 - 3y + z = 2$

$3.1x + 2.2y - 3z = 0$ $\qquad$ $3x + \sqrt{2}y = 6$ $\qquad$ $\cos(x) + y - z = 6$

(Various Sections)

Match the following with the matching term from those listed below

________________ 4. A matrix that has zero entries, except on the main diagonal (where entries are 1.)
________________ 5. A system of linear equations whose right hand side is all 0's.
________________ 6. The matrix obtained from a matrix by interchanging its rows and columns.
________________ 7. A matrix with zero entries ***except on*** the main diagonal.
________________ 8. A matrix whose inverse exists.
________________ 9. A matrix with the same number of rows and columns.
_______________ 10. A matrix A such that $A^t = A$.

diagonal	identity	reduced echelon	transpose
elementary	interchange	square	trivial
homogeneous	invertible	symmetric	

Sections 1.1 and 1.2

11. Which of the following is **NOT** an elementary row operation?
a) Interchange two columns.
b) Multiply a row by a nonzero number.
c) Interchange two rows.
d) Add a multiple of one row to another row.

12. $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad$ $\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 3 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 3 & 2 & 0 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

Which of the above matrices are in reduced row echelon form?

13. For the following equations representing two lines,

$$2x + 4y = 8$$
$$2x - y = 1$$

a) The graphs of the equations are (circle correct)
 i) Two intersecting lines
 ii) Two parallel lines, non-intersecting
 iii) Two lines that are coincident (i.e. lie on top of each other).

b) It will have (one solution, no solution, many solutions). (Circle correct answer.)

14. If you were performing row reduction on the following matrix to bring it into ***reduced row echelon*** form, write down what the ***next step*** would be and the resulting matrix

a) $\begin{pmatrix} 1 & 0 & 4 & -1 \\ 0 & 1 & 2 & -3 \\ 0 & -2 & 3 & 5 \end{pmatrix}$ b) $\begin{pmatrix} 0 & 0 & 4 & -1 \\ 2 & 1 & 2 & -3 \\ 3 & -2 & 3 & 5 \end{pmatrix}$

15. Suppose the augmented matrix form for a system of equations has been reduced to the following form:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Write down the general solution, if any, assuming unknown variables x_1, x_2, and x_3.

16. a) Write the down the augmented matrix for the following system of linear equations:

$$\begin{aligned} x_1 - x_2 + 3x_3 &= 3 \\ 2x_1 - x_2 + 2x_3 &= 2 \\ 3x_1 + x_2 - 2x_3 &= 3 \end{aligned}$$

b) Solve the system using Gauss-Jordan elimination on the above matrix. Show all work.

Section 2.1, 2.2

17. Which of the following are true properties of matrices (Circle)
(Assume A,B, and C are matrices of suitable dimensions, c is a scalar).

A + B = B + A　　c(A + B) = cA + B　　A(BC) = A(BC)
c(AB) = (cA)B　　AB = BA　　A(B + C) = AB + BC

18.

A = $\begin{pmatrix} 1 & 4 & 1 \\ -2 & 3 & 0 \\ 1 & 0 & 4 \end{pmatrix}$ B = $\begin{pmatrix} 2 & -3 & 0 \\ 1 & 0 & 6 \\ 0 & 3 & 1 \end{pmatrix}$ C = $\begin{pmatrix} 5 & 1 & 9 & 0 \\ 4 & 0 & 6 & -2 \\ 3 & -1 & 2 & 4 \end{pmatrix}$

If defined calculate the following (If not defined, write down not defined and explain why):
a) A + 2B　b) AB　c) CA　d) tr(A)　f) A^t　e) A^2

19. Write the down the matrix form for the following system of linear equations:

$$\begin{aligned} x_1 - x_2 + 3x_3 &= 3 \\ 2x_1 - x_2 + 2x_3 &= 2 \\ 3x_1 + x_2 - 2x_3 &= 3 \end{aligned}$$

20. T F The set of solutions to a homogeneous system of is closed under addition and scalar multiplication.

21. Prove the following: Let A and B be symmetric matrices. AB is symmetric if and only if and AB = BA

Section 2.4

22. Calculate the inverse (if it exists) for the following matrix.
If it doesn't exist explain why. Show all of your work (you may use your calculator only to check your answer, not compute it!)

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

23. a) Complete the following: If A is invertible, then AX = B has a unique solution and is given by $X = A^{-1}B$ or $X = B^{-1}A$ (circle correct).

b) Prove the statement in part a) using the definition of inverse and properties of matrix multiplication.

c) Prove the following: If A is invertible, then its inverse is unique.

Section 3.1 Introduction to Determinants

24. Let $A = \begin{bmatrix} -3 & 1 & -2 \\ 2 & 4 & 5 \\ -1 & 0 & 2 \end{bmatrix}$ Compute a) The minor $M_{2,1}$ b) The cofactor $C_{2,1}$

25. Determine the determinant of the following matrix using as little work as possible – but show that work!

$$\begin{vmatrix} 8 & 7 & -1 & 2 \\ 4 & 2 & 0 & 1 \\ 0 & 3 & 0 & 2 \\ 0 & 5 & 0 & -2 \end{vmatrix}$$