M241 Probability Review Guide for 3rd Exam Semester 011

Chapter 4 Continuous Distributions

Know what a continuous random variable is.
Important examples of continuous random variables and their distributions:
Normal Distribution
Uniform on an interval (a,b)
Exponential Distribution

Section 4.1 Probability Densities

Definition of density function f(x) for a continuous random variable

- Interpretation of P[a <= X <= b] as the area under the curve f(x) between a and b and as the integral of f(x) over that interval -- be able to calculate for a simple *f(x).*
  Definition of E(X) for X continuous random variable with density *f(x)* – be able to apply to calculate E(X) given f(x)
- Definition of E(g(X)) where g is a function -- apply to examples.
- Be able to calculate Var(X) and SD(X) for continuous r.v. X with given density f(x)
- Be able to work exercises like those assigned, including exercise 3 page 275.
- Know P[X = x} is always 0 for cont r.v. X and P[X is in a small interval *dx*] is approximately *f(x)*dx*
- Independence of continuous random variables is defined the same way as for discrete random variables.
- Important Continuous Distributions

The Uniform Distribution on the interval (a,b)

Know what its density is, be able to calculate its mean, variance, and standard deviation, and P[c<X<d]

The Normal Distribution

We have already studied this.

Recognize the standard density function for the Standard Normal Distribution (Normal(0,1) and the for Normal Distribution with mean μ.
Know that if X is Normal (μ,σ^2), the standardized $X^* = (X-\mu)/\sigma$ is Normal(0,1).
Be able to apply to examples like the exercises assigned (6, 7, and 8)

Section 4.2 Exponential and Gamma Distributions (through page 289)

Know how to describe a Poisson Arrival Process (see page 289 # 1)
Know that number of arrivals in time t has Poisson(lt) distribution; Waiting times between arrivals are independent with exponential(l) distribution (2 and 3 page 289); and Waiting tim until rth arrival has gamma(r,l) distribution (4, page 289)

Know what the density function is for the **exponential(λ) distribution**.
Give some examples of random events which are modeled with an exponential distribtution.
Know the survival function P(T>t), E(T) and SD(T) for an exponential r.v T.
Know what the **memoryless property of the exponential distribution** says. Be able to apply to problems.
Be able to work problems involving a Poisson arrival process like example 3 pag 284-5 and assigned exercises from this section.
Gamma Distribution: Know that $T_r = W_1 + \ldots + W_r$ is the waiting time for rth event in a Poisson process and has Gamma Distribution. Don't have to memorize its density function. Do need to know how to calculate its right tail probability (see page 286 and example 3 part f) as well as assigned exercise 5)

Section 4.5 Cumulative Distribution Functions (to page 319)

For continuous random variables, know the relationship between density f(x) and c.d.f. F(x): *f(x)* is the derivative of F(x) and F(x) an antiderivative (indefinite integral) of f(x).
For simple density functions f(x) (including the uniform and the exponential) calculate the c.d.f F(X).

$$F(x) = \sum_{y \le x} P(X = y)$$

For a continuous random variable X with density *f(x)*

$$F(x) = \int_{-\infty}^{x} f(x)dx$$

Be able to sketch the c.d.f for both continuous and discrete r.v.'s
Know that the distribution function of X_{max}, the maximum of independent $X_1, \ldots, X_n$ is
$F_{max}(x) = F_1(x)F_2(x)\ldots F_n(x)$
and the distribution function of X_{min}, the minimum of independent $X_1, \ldots, X_n$ is
$F_{min}(x) = 1 - (1 - F_1(x))(1 - F_2(x))\ldots(1 - F_n(x))$
Applying to the minimum of independent exponential random variables (example 3, page 317 and assigned exercises)
Application to circuits like example 4, page 317 - 318 and assigned exercise 8

Chapter 5
Know that for joint distribution of a pair of continuous random variables X and Y, we are interested in P[(X,Y) is in some set B], where **B is a subset of the plane.**

Section 5.1 Uniform Distributions
Know that the joint distribution of independent uniformly distributed random variables X and Y is uniform on the on the rectangle.
Use **methods of plane geometry** or **simple calculus** to calculate probabilities of functions g(X,Y) where X and Y are independent uniform random variables and conditional probabilities as in examples 1, 2, 3, and exercises 1, 3, 4, and 6a.

Section 5.2 Joint Density Functions (through page 351 example 2)
Know the definition of the joint density function for a pair of random variables X and Y

$$P((X,Y) \in B) = \iint_B f(x,y)dx\,dy$$

For this to be a valid density function it must satisfy

f(x,y) ≥ 0 and $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dx\,dy = 1$

Know that the geometric interpretation of this -- probability that (X,Y) is in B is the volume under the surface f(x,y) over the region B.

Definition of the marginal density functions:

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy \qquad f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx$$

For **independent** random variables, know that the joint density *f(x,y)* is just the product of the marginal densities $f_X(x)\, f_Y(y)$
Know how to calculate the expectation of a function g(X, Y) – especially E(XY)
Study Examples 1 and 2 and be able to work problems like them (as well as exercises 3, 4, and 5)