M241 Probability Exam # 3 Practice Questions

1. a) Given a ***density function f(x)*** for a random variable X, the probability
$P(a \le X \le b)$ = ___________________________

b) Draw a sketch of what this means for a given density function f(x) and an interval (a,b). (Shade area that represents $P(a \le X \le b)$)

c) Write down the **formula** for the expectation E(X) in terms of the density function *f*(x).

d) Write down the formula for the cumulative distribution function F(x) in terms of *f*(x).

2. Suppose that X is a random variable whose density is

$$f(x) = \frac{c}{x^3}, \text{ for } x > 1, \text{ and } f(x) = 0, \text{ otherwise}$$

Find a) c b)P[2 < X < 4] c) E(X) d) $E(X^2)$ e) Var(X) f) The cumulative distribution function F(*x*) for X.

3. Answer for the following properties of a Poisson(λ) Arrival Process.

a) T F Events of arrivals over disjoint intervals are independent.

b) The average rate of arrivals per unit of time is __________

c) The number of arrivals in time *t* has an ____________distribution.

d) P(arrival in a sufficiently small interval Δt) is approximately _______

e) The waiting times W_1, W_2, … between arrivals are each independent with ___________________ distribution

f) The waiting times T_r until the *r*th arrival has _____________ distribution.

4. Suppose people are arriving at a bank at an average rate of 20 per hour according to a Poisson arrival process. Find:

a) The probability that the fifth person arrives within 5 minutes after the fourth person.

b) The probability that the third person arrives by time 15 minutes.

c) The expected time at which the first person arrives.

d) The expected time that the third person arrives.

e) The probability that exactly 2 people arrive during the first minute.

5. If X is a Normal random variable with mean μ and variance σ^2, then random standardized random variable X* is given by

$a)\quad \frac{X-\mu}{\sigma^2}$

$b)\quad \frac{X-\mu}{\sigma}$

$c)\quad \frac{X-\sigma\mu}{\sigma^2}$

$d)\quad \frac{X-\sigma\mu}{\sigma}$

6. The density function for the standard Normal (0,1) random variable X* is given by

$a)\quad e^{-\frac{x^2}{2}}$

$b)\quad e^{\frac{x^2}{2}}$

$c)\quad e^{-\frac{x}{2}}$

$d)\quad \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

e) none of the above

7. Measurements on the weight of a rock are independent and identically distributed with a mean of 10 grams and a standard deviation of 1 gram.

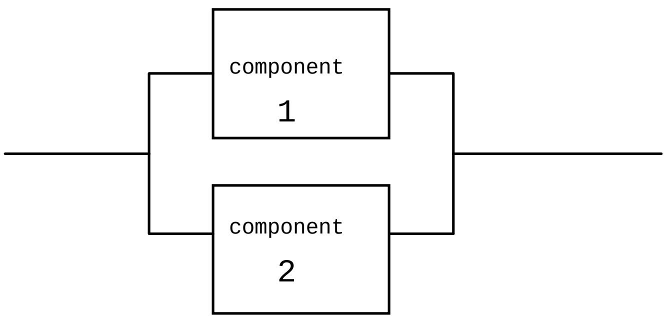
a) Find the probability that a measurement taken will be between 9 and 10.2 grams if we assume that the individual measurements are normally distributed.
b) Find probability that the mean (average) of 100 measurements will fall between 9 and 10.2 grams.

8. a) The following

$$f(x) = \begin{cases} 0.2e^{-.2x} & \text{if } x \geq 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

is the density function for the ____________________ distribution with parameter λ = _____.
b) Calculate its cumulative distribution function F(x)

9. Suppose we have electrical components whose lifetimes are exponentially distributed with parameter λ = 1/5 hours (and hence mean $1/\lambda$ = 5 hours).
a) Find the probability that a component survives at least 6 more hours.
b) Does you answer for a) change if you know that the component has lasted 3 hours already? (Explain)
c) Suppose we connect two of these components in parallel . What is the probability that the circuit comprised of these two parallel components **survives at least** 12 hours.



10. Suppose X is a random variable with **Uniform(0,2)** distribution.
a) Calculate the probability P[1 < X < 2]
b) Write down the density function of X
c) Suppose X and Y are independent and both Uniform(0,2) (so their joint distribution is uniform on the square [0, 2] x [0, 2] Calculate the probability P(|X-Y| <= 1/2). (It should help you to sketch the region |X-Y| <= ½ inside the square first.)

11. Let the joint density of X and Y be given by

$$f(x,y) = \begin{cases} e^{-(x+y)}, & \text{x} > 0, \text{y} > 0 \\ 0, & \text{otherwise} \end{cases}$$

a) Calculate the marginal density of *f* with respect to X, $f_X(x)$
(Note that $e^{-(x+y)} = e^{-x} e^{-y}$)
b) Calculate the marginal density of *f* with respect to Y, $f_Y(y)$
c) Are X and Y independent random variables? (Yes, No) Justify your answer:
d) Calculate P(X < 2Y)