MTH 236 Calculus IV Fourth Exam 092 Name ____________________________

(8 pts) 1. Sketch the region of integration represented by the following integral. Then evaluate by converting to polar coordinates.

$$\int_0^2 \int_0^{\sqrt{4-y^2}} x^2 + y^2 \, dx \, dy$$

2. Interpret the physical meaning of each of the following:

(4 pts) a) lamina

(4 pts) b) center of mass of a lamina

(8 pts) 3. A lamina has the shape of a closed region bounded by the graphs of $y = x^2$, $y = x$. It has the density function $\rho(x, y) = x^2$. Write down the iterated integral in rectangular coordinates for the ***moment of inertia about the x-axis***. (Do not evaluate).

(8 pts) 4. a) Write down the double integral formula for the surface area of that portion of a surface $z = f(x,y)$ that lies over the region R.

b) Outline the justification of that formula as a limit of the appropriate Riemann sum, including in your explanation how the cross product formula for area of a parallelogram is used.

(10 pts) 5. Set up the iterated integral for the surface area of that portion of the surface $z = 4 - x^2 - y^2$ that lies over the region R in the x-y plane defined by
R = $\{(x,y)$: $x^2 + y^2 \le 4\}$. Do not evaluate either integral.

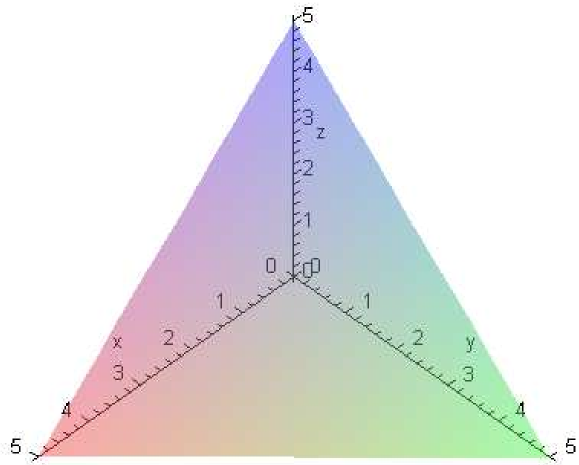
a) In rectangular coordinates

b) In polar coordinates

(8 pts) 6. Evaluate the following triple integral. Show each step

$$\int_0^{\pi/2}\int_0^{y/2}\int_0^{1/y} \sin(y)\, dz\, dx\, dy$$

(8 pts) 7. Set up a triple integral to find the volume of the solid bounded by the plane $z = 5 - x - y$ that is in the first octant (i.e. x, y, and z all positive) *(as plotted below)* . (Do not evaluate).



(8 pts) 8. Set up the triple integral in cylindrical coordinates to find the volume of the solid Q described by

$$Q = \{(x, y, z) : 0 \le z \le 9 - x - 2y, x^2 + y^2 \le 4\}$$

(Do not evaluate).

(8 pts) 9. Convert the following triple integral from rectangular to spherical coordinates (do not evaluate).

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \sqrt{x^2 + y^2 + z^2}\, dz\, dy\, dx$$

Formulas for Center of Mass and Moments of Inertia for Lamina

Mass :

$$m = \iint_R \rho(x, y)dA$$

Moments of Mass :

$$\mathrm{M}_{\mathrm{x}} = \iint_R y\rho(x, y)dA$$

$$\mathrm{M}_{\mathrm{y}} = \iint_R x\rho(x, y)dA$$

Center of Mass :

$$(\bar{x},\bar{y}) = (\frac{M_y}{m}, \frac{M_x}{m})$$

Moments of Intertia

$$\mathrm{I}_{\mathrm{x}} = \iint_R y^2\rho(x, y)dA$$

$$\mathrm{I}_{\mathrm{y}} = \iint_R x^2\rho(x, y)dA$$