MTH 236 Calculus IV Test 2 Semester 092

Name _____

(5 pts) 1. Show that the following limit does not exist. Give a clearly written explanation.

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^3+y^3}$$

(3 pts) 2.
The partial derivative of f **with respect to** y is defined by
$f_x(x,y) =$

a) $\lim_{\Delta y\to 0} \frac{f(x+\Delta y, y+\Delta y) - f(x,y)}{\Delta y}$

b) $\lim_{\Delta y\to 0} \frac{f(x,y) - f(\Delta x, \Delta y)}{\Delta y}$

c) $\lim_{\Delta y\to 0} \frac{f(x,y+\Delta y) - f(x,y)}{\Delta y}$

d) $\lim_{\Delta y\to 0} \frac{f(x,\Delta y) - f(x,y)}{\Delta y}$

(3 pts) 3. The geometric interpretation of the partial derivative $f_y(1,2)$ for a function $f(x,y)$ is

a) The rate of change in the direction of the vector $\mathbf{u} = \mathbf{i} + 2\mathbf{j}$
b) The slope of the normal line to tangent plane for surface z = $f(x,y)$ at the point (1,2).
c) The direction in which the function's coordinates are increasing most rapidly from the point (1,2).
d) The slope of the tangent line to the surface in the direction of the y axis.

(4 pts) 4. Complete the (formal) definition of differentiability for a function of two variables. Let $\Delta z = f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0)$. Then $z = f(x,y)$ is differentiable at (x_0,y_0) if

(3 pts) 5. Which of the following is a sufficient condition for differentiability of a function *f* of the variables *x* and y, in an open region R in terms of the function's partial derivatives?

a) f_x and f_y both exist in R
b) f_x and f_y both exist and are differentiable in R
c) f_x and f_y both exist and are continuous in R
d) f_x and f_y both exist and are not equal to 0 in R

(6 pts) 6. a) Write down the equation for the total differential *dz* for the function $z = \ln(x^2+y^2)$.

b) Use the differential *dz* to approximate the change in $z = \ln(x^2+y^2)$ as (x,y) moves from (0,1) to (0.01, 0.98). Show your work.

(6 pts) 7. Use the Chain Rule to calculate $\frac{\partial z}{\partial t}$ for $z = x^3 y$ and $x = 3t, \quad y = t^2$ Show work. Express your answer in terms of *t*.

(6 pts) 8. Solve using the Chain Rule: . The height of a right circular cylinder is **increasing** at a rate of 0.5 centimeters per second and the radius of the base is **decreasing** at the rate of 0.2 centimeters per second. Find the rate at which the volume is changing when the radius of the base is 10 centimeters and the height is 40 centimeters.
($V = \pi r^2 h$)

(12 pts) 9. Let $f(x, y) = y^2 - x^2$

a) Calculate the gradient vector ∇f at the point (1, 1).

b) Calculate the directional derivative of *f* in the direction of the vector **v** = **i** + 2**j** at the point (1, 1).

c) Find the direction in which the function *f* is increasing most rapidly at the point (1, 1).

d) Find the maximum directional derivative at the point (1,1).

(3 pts) 10. Complete: The gradient vector of the function $f(x,y)$ at the point (x_0, y_0) is normal to the _____

a) tangent plane to the surface $z = f(x,y)$ at the point (x_0, y_0).

b) all directional derivatives on the surface $z = f(x,y)$ at the point (x_0, y_0).

c) the level curves of the surface $z = f(x,y)$ at the point (x_0, y_0).

d) tangent lines to the surface $z = f(x,y)$ at the point (x_0, y_0).

(10 pts) 11. a) Find an equation of the tangent plane to the surface $x^2 + y^2 + 2z^2 = 23$ at the point (1, 2, 3). Show all work.

b) Find the parametric equations of the normal line to the same surface at the same point.

(10 pts) 12. a) Find the first and second order partial derivatives indicated for the following function: $f(x, y) = 3x^3 + y^2 - 9x + 4y$

$f_x(x,y)$ $f_y(x,y)$

$f_{xx}(x,y)$ $f_{yy}(x,y)$

$f_{xy}(x,y)$

b) Use the second derivative test to find find and classify critical points *f* or the above function as relative maximum, minimum or saddle points. Show all work clearly.