MTH 236 Calculus IV First Exam Semester 092 (Chapter 12, 13.1, 13.2)

Name ___________________________________

(4 pts) 1. Calculate the following limit, showing your work.

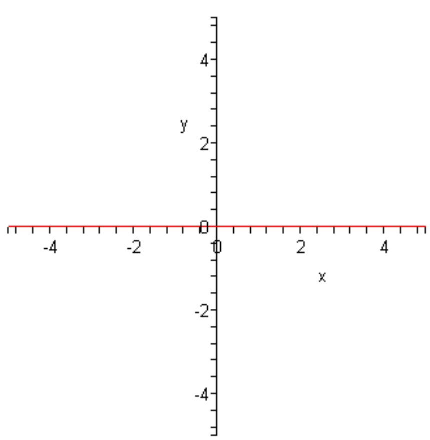
$$\lim_{t\to 0}\left(\frac{2\sin(t)}{t}\mathbf{i}+(t-1)\mathbf{j}+\frac{3t^2-2t}{t^2}\mathbf{k}\right)$$

(6 pts) 2. $\mathbf{r}(t)=\sqrt{3-t}\,\mathbf{i}+t\mathbf{j}$

a) Find the domain for the above vector-valued function **r**.

b) Give the equation in rectangular coordinates for the plane curve represented by this vector valued function.

c) Sketch the graph of this plane curve below, showing its orientation:



(6 pts) 3. Given that

$\mathbf{r}'(t) = 2t\mathbf{i} + e^{t}\mathbf{j} + e^{-t}\mathbf{k}$

and

$\mathbf{r}(0) = \mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$

Find **r**(t)

(9 pts) 4. 4. Suppose that a spear is thrown from a height of **6 feet above ground** and is thrown an angle of 60 degrees with speed v_0 of 90 feet per second. Assume gravity is 32 feet per second^2**).** Recall that the position vector **r**(t) is given by

$$\mathbf{r}(t) = [(v_0 \cos\theta)t]\mathbf{i} + [h + (v_0 \sin\theta)t - gt^2/2]\mathbf{j}$$

a) Calculate the time *t* when the spear reaches it maximum height.

b) Calculate its coordinates at this time -- i.e. what is its maximum height, and what is the horizontal distance from the launch point when it reaches its maximum height.

(20 pts) 5. Calculate for the following position function
$\mathbf{r}(t) = 4\sin t\,\mathbf{i} + 3t\,\mathbf{j} + 4\cos t\,\mathbf{k}$

a) Velocity vector **v**(t)

b) Speed at time t

c) Acceleration vector **a**(t)

d) Unit tangent vector **T**(t)

e) Principal unit normal vector **N**(t)

f) curvature K at t.

g) Tangential component of acceleration, $\mathbf{a_T}$.

h) Normal component of acceleration, $\mathbf{a_N}$.

i) Find the arc length of the curve for over the interval [0,π].

(3 pts) 6. Which of the following is the correct interpretation of the unit tangent vector for a curve representing a particle in motion?

a) It is perpendicular to the acceleration vector.
b) It represents the speed of the particle.
c) It points in the direction of the concavity of the curve representing the particle's motion.
d) It points in the direction of the motion of the particle at time t.

(3 pts) 7. Which of the following is the correct interpretation of the principal unit normal vector N for a given curve?

a) It is in the direction of the tangent line to the curve.
b) It is normal to the curve and points towards the concave side of the curve.
c) It is the derivative of the velocity vector.
d) It is normal to the velocity vector and points in the direction of acceleration.

(3 pts) 8. The curvature K of a curve measures:

a) the energy expended in the direction of the principal unit normal N.
b) how sharply a curve bends.
c) the rate of change of the direction of motion with respect to time t.
d) the energy expended in the direction of the unit tangent vector T.

(3 pts) 9. Suppose a curve has a **normal** component of acceleration which is always equal to 0 – what can you say about the curve.

a) It has constant speed.
b) The radius of the circle of curvature is constant.
c) It is moving in a straight line.
d) It is turning with an amount proportional to the change in speed.

(3 pts) 10. The formula for the arc length parameter s as a function of t is given by

a) $s(t) = \int_a^t \mathbf{v}(u)du$ b) $s(t) = \int_a^t \|\mathbf{r}(u)\|du$

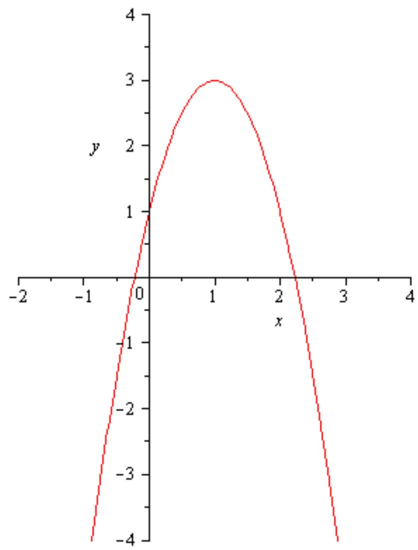
c) $s(t) = \int_a^t \|\mathbf{v}(u)\|du$ d) $s(t) = \int_a^t \mathbf{r}(u)du$

(3 pts) 11. If we apply the Second Fundamental Theorem of Calculus to the formula above, we get

a) $\frac{ds}{dt} = \|\mathbf{v}(t)\|$ b) $\frac{ds}{dt} = \mathbf{a}(t)$.

c) $\frac{ds}{dt} = \|\mathbf{a}(t)\|$ d) $\frac{ds}{dt} = \mathbf{v}(t)$

(8 pts) 12. Below is a graph of the curve $y = -2\,(x-1)^2 + 3$



a) Compute the curvature K for this at the point (1,3). Show work.

b) Compute the radius of the circle of curvature at this point.

c) ***Sketch the circle of curvature at (1,3) on the above graph – be precise.***

(5 pts) 13. a) Find the domain of the function $f(x, y) = \ln(3x + 2y)$

b) Evaluate $f(2, 3)$

(4 pts) 14. Describe the level curves of the surface represented by the function $f(x, y) = 2y^2 + 3x^2$.