Review Guide and Practice Test 4th Test Calc IV -- Test Monday April 26th

Section 14.3: Polar Coordinates
Apply Theorem 14.3 to setting up and evaluating double integrals in polar form – including converting from rectangular to polar, and finding volume under a surface (or between two surfaces).
Practice Problems: Exercises 9, 11, 15, 27, 43, 44

0. Use polar coordinates to evaluate $\iint_R \sqrt{x^2 + y^2}\, dA$ where the region R is region between the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

Section 14.4 Center of Mass and Moments of Inertia.

You do not need to remember the formulas for anything other than mass and center of mass.
You should be able to interpret the physical meaning of:
Lamina
Mass
Center of Mass
Second Moment (Moment of Inertia) about a line (e.g. x axis or y axis)
Kinetic Energy -- How moment of inertia of revolving lamina can estimate kinetic energy.
Radius of gyration of a revolving mass about *x* or *y* axis.

Evaluate these for a given density and region describing a lamina.
1. Explain the physical interpretation of
a) lamina
b) mass
c) center of mass
d) etc. (as above)

Practice Problems:
2. A lamina has the shape of a closed region bounded by the graphs of $x + y = 3$, $3x + y = 3$, and $y = 0$. It has the density function $\rho(x, y) = 2xy$.
Find a) mass b) the coordinates for the center of mass c) moment of inertia about the *x* axis. Show work.

Section 14.5: Surface Area
Outline the development of the formula for surface area of a surface over a region (see page 1017)
Set up and evaluate the double integral in either rectangular or polar coordinates to find surface area of given surface over a region.
Use formula to set up and evaluate the appropriate double integral to find surface area.
Practice Problems: Exercises 5, 11, 15

3 a) Write down the double integral formula for the surface area of that portion of a surface z=f(x,y) that lies over the region R.
b) Outline the justification of that formula as a limit of the appropriate Riemann sum, including in your explanation how the cross product formula for area of a parallelogram is used.

4. Find the surface area of that portion of the surface $z = y + \frac{x^2}{2}$ that lies over the square region in the xy plane having vertices (0,0), (1,0), (0,1), and (1,1).

5. Find the surface area for the portion of the surface $z = 1-x^2 - y^2$ that lies above the xy-plane.

Section 14.6 Triple Integrals and Applications
Know how to evaluate a triple integral in any particular order of integration
Know the definition of the triple integral
Use triple integrals to find the volume of a solid region
Use triple integrals to find Center of Mass and Moments of Inertia (only need to know mass formula -- I will give the rest)
Practice problems 13, 16, 22, 23, 35

6. Use a triple integral to find the a) volume and b) mass c) Center or Mass of a solid Q in the first octant bounded by the cylinder $x^2 + y^2 = 4$, and the plane $2y + z = 4$ where the density of this solid is given by $\rho(x, y, z) = x$.

Section 14.7 Triple Integrals in Cylindrical Coordinates and spherical coordinates
Be able to set up and evaluate triple integrals in cylindrical and (for simple example) spherical coordinates.
Convert a rectangular triple integral to cylindrical and spherical.

Practice Problems 15, 16, 19, 22, 33

7. Use cylindrical coordinates to find the volume of the solid in the first octant that is bounded by the cylinder $x^2 + y^2 = 2y$
and the top part of the cone $z^2 - x^2 - y^2 = 0$

8. Convert the following integral from rectangular to spherical coordinates
$$\int_{-4}^{4}\int_{0}^{\sqrt{16-x^2}}\int_{0}^{\sqrt{16-x^2-y^2}} (x^2 + y^2 + z^2)\,dz\,dy\,dx$$

14. a) Convert the triple integral $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{(x^2+y^2)/2}(x^2+y^2)dz\,dy\,dx$ to cylindrical coordinates

b) Evalute the integral in part a) Show work for ***each step*** in the evaluation of the integral.