

Review Guide and Practice Test 3rd Test Calc IV -- Test Friday April 17th Covered parts of Chapters 13, 14

Section 13.10 Lagrange Multipliers

Using Lagrange Multipliers, set of the necessary equations and solve them to find the maximum or minimum of a function of two variables subject to one or two constraints.

Practice Problems: Exercises 5, 13, 15, 17, 19

1. Use Lagrange multipliers to minimize $f(x, y, z) = 4x^2 + y^2 + z^2$ with the constraint that $2x - y + z = 4$.
2. Set up the equations using the method of Lagrange Multipliers with two constraints:
Find the point on the intersection of the plane $x + 2y + z = 10$ and the paraboloid $z = x^2 + y^2$ that is closest to the origin. (Recall that it is easier to minimize the square of the distance).
3. Set up the equations using the method of Lagrange Multipliers with two constraints (Do not solve – just write down the five equations that need to be solved):
Find the maximum value of $f(x, y, z) = x + 2y + 3z$ that is constrained to lie on the intersection of the plane $y + z = 1$ and the cylinder $x^2 + y^2 = 2$.

Chapter 14

Section 14.1 Iterated Integrals, Area in Plane

Evaluate an iterated integral

Given a region in the plane, set up an iterated and evaluate an iterated integral to find the area of the region.

Given an iterated integral, sketch the region, and set up the integral with the order of integration reversed.

Practice Problems: Exercises 13, 29, 33, 61

4. a) Sketch the region R of integration represented by the integral $\int_{-1}^1 \int_0^{1-x} (4 - y) dy dx$.
b) Write down what the iterated integral would be if the order of integration is reversed (do not need to evaluate).
5. Set up the iterated integral representing the area of the region R bounded by the graphs of $y = 2x$ and $y = x^2$, using the order $dy dx$. (Do not need to evaluate).

Section 14.2 Double Integrals and Volume

Give the definition of a double integral as a limit of a Riemann sum and explain its interpretation in terms of the volume of a solid approximated by the sum of volumes of approximating rectangles.

Using Fubini's Theorem, solve problems to find volume setting up and evaluating the appropriate double integral.

Practice Problems: Exercises 13, 19, 23, 27, 49

6. Use a double integral to find the volume in the first octant under the surface $f(x, y) = x^2 y^2$, and above the region in the plane bounded the lines $y = 1$, $y = 2$, $x = 0$, and $x = y$
Sketch the region in the plane first.
7. Give the complete and precise definition of a double integral as a limit of a Riemann sum and explain its interpretation in terms of the volume of a solid approximated by the sum of volumes of approximating rectangular solids.

Section 14.3: Polar Coordinates

Explain how the formula for polar coordinate form for a double integral is developed using polar partitions and approximating sums.

Apply this formula (Theorem 14.3) to setting up and evaluating double integrals in polar form – including converting from rectangular to polar, and finding volume under a surface (or between two surfaces).

Practice Problems: Exercises 9, 11, 15, 27, 43, 44

8. Explain how the formula for polar coordinate form for a double integral is developed using polar partitions and approximating sums.

9. Use polar coordinates to evaluate $\iint_R \sqrt{x^2 + y^2} dA$ where the region R is region between the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

Section 14.4: Center of Mass and moments of Inertia.

I will give you the formulas for moments of mass, center of mass, moments of inertia, and radius of gyration.

Interpret the physical meaning of lamina, center of mass, moment of inertia, radius of gyration.

Compute these for given laminas and densities.

Practice Problems: Exercises 7, 11, 31, 35

10. Interpret the physical meaning of each of the following:

- a) lamina
- b) center of mass
- c) moment of inertia

11. A lamina has the shape of a closed region bounded by the graphs of $x + y = 3$, $3x + y = 3$, and $y = 0$. It has the density function $\rho(x, y) = 2xy$.

Find a) the coordinates for the center of mass. b) moment of inertia about the x axis. Show work.

Section 14.5: Surface Area

Outline derivation of the double integral formula for surface area of $z = f(x,y)$ over region.

Use formula to set up and evaluate the appropriate double integral to find surface area.

Practice Problems: Exercises 5, 11, 15

12. Outline the derivation of the formula for surface area over a region R.

13. Find the surface area for the portion of the surface $z = 1 - x^2 - y^2$ that lies above the xy -plane.