

Review Guide 2nd Test Calc IV -- Test Friday March 5th
Chapter 13 Functions of Several Variables

Section 13.1

- Understand concept of a function of several variables Evaluate a function of several variables at given value(s). (Exercise 5)
- Determine domain of a given function of several variables. (Exercise 22)
- Know definition of level curves for a function $f(x,y)$ – describe for a given function (Exercise 51, 56)

1. a) Find the domain of the function $f(x, y) = \sqrt{32 - 4x^2 - y^2}$
b) Evaluate $f(1, 2)$

2. Describe the level curves of the surface represented by the function $f(x, y) = \ln(y - x^2)$.

Section 13.2 Limits and Continuity

- Understand definitions and interpretations of limits and continuity applied to functions of several variables.
 - Determine the limit of a function at a point.
 - Determine whether a function is continuous at a point.
 - Apply to problems (Exercises: 9, 13, 20, 26)
3. Show that the following limit does not exist. Give a clearly written explanation.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$$

Section 13.3 Partial Derivatives

- Definition of partial derivative as a limit.
 - Calculate partial derivatives for given function
 - Calculate second-order partial derivatives.
 - Interpretation of partial derivations as slopes in x and y direction of surface.
 - **Practice Problems: 13, 33, 29, 30**
4. The slope of the tangent line to the surface $z = f(x,y)$ at the point $(1,1)$ in the direction parallel to the y axis is given by:
- a) The directional derivative in the direction of $\mathbf{u} = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$
- b) The gradient vector at the point $(1,1)$
- c) The partial derivative $f_y(1,1)$
- d) The point on the tangent plane to $z = f(x,y)$ with (x,y) coordinates $(0,1)$.
5. Give the (formal) definition of the partial derivative $f_x(x,y)$ of the function $f(x,y)$.

Section 13.4 Differentials

- Definition and computation of total differential
- Definition of differentiability
- Statement of Theorem 13.4

- How to use the differential to approximate the change in a function $f(x,y)$ as (x,y) moves to a nearby point (like example 3).
- How to use the total differential to determine error bounds in measurements (like example 4 and assigned exercises).
- **Practice Problems: 1, 6, 32, 36, 40**

6. Complete the (formal) definition of differentiability for a function of two variables. Let $\Delta z = f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0)$. Then $z = f(x,y)$ is differentiable at (x_0, y_0) if

7. Give a sufficient condition for differentiability in terms of a function's partial derivatives.

8. The volume of a right circular cone is given by $V = \frac{\pi r^2 h}{3}$, where r is the radius of the base and h is the height. The radius of the base of the cone is 12 centimeters and height is 16 centimeters. The possible error in each measurement is $\pm .02$ centimeters. Use differentials to approximate the maximum possible error in computing the volume.

Section 13.5 Chain Rule for Functions of Several Variables

- Use the chain rule to calculate derivatives of composite functions of several variables.

(Like examples 1, 4, and 5 and problems like those assigned)

- Apply the chain rule to related rates problems (like 51-54)
- **Practice Problems 2, 5, 15, 24, 51**

9. Use the Chain Rule to calculate $\frac{\partial w}{\partial s}$ for $w = x^2 + 2y^3$ and $x = s + t$, $y = t - s$. Show work. Express your answer in terms of s and t .

10. Solve using the Chain Rule: The length, width and height of a rectangular solid are increasing at the rate of 3 feet per minute, 2 feet per minute and $\frac{1}{2}$ foot per minute, respectively. Find the rate at which the volume is changing at the instant the length is 10 feet, the width is 6 feet, and the height is 4 feet. ($V = lwh$.)

Section 13.6 Directional Derivatives and Gradients

- Know how to calculate the gradient vector of a function of two or three variables.
- Know how to calculate the directional derivative of a function of two or three variables.
- What the directional derivative means in terms of rate of change of the function in a particular direction
- Apply the properties of the gradient vector to determine the direction and magnitude of most rapid increase, decrease for a function of two or three variables.
- Know statement of Theorem 12.12 -- gradient vector is normal to level curves.
- **Practice problems: 2, 9, 15, 23, 35, 42, 44, 65 – 70.**

11. Let $f(x, y) = \ln(x^2 + y^2 + 1)$

- a) Calculate the gradient vector ∇f at the point $(1, -2)$.
- b) Calculate the directional derivative of f in the direction of the vector $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$

at the point (1, -2).

- c) Find the **direction** in which the function f is increasing most rapidly at the point (1, -2)
- d) Find the maximum directional derivative at the point (1,-2).
- e) Find the **direction** in which the function f is decreasing most rapidly at the point (1, -2) and also find the maximal amount by which it is decreasing.

12. T F The gradient vector for the surface $z = f(x,y)$ is tangent to the level curves of the surface.

Section 13.7 Tangent Planes and Normal Lines

- Find the normal vector, normal line, and tangent plane to a surface $f(x,y,z) = 0$ at a point by using the gradient vector at that point.
- Find the **parametric and symmetric** equations for the tangent line to the curve of intersection of two surfaces.
- **Practice Problems: 7, 21, 34, 41**

13. a) Find an equation of the tangent plane to the surface $x^3y - y^2 + z^2 = 7$ at the point (1, 2, 3). Show all work.

b) Find the parametric and symmetric equations of the normal line to the same surface at the same point.

14. Find the **parametric and symmetric** equations for the tangent line to the curve of intersection of the two surfaces given by $z = x^2 + y^2$ and $x + 2y - z = -5$ at the point (2,3,13).

Section 13.8 Extreme Values

- Given a function of two variables, calculate the critical points and determine which if any are relative and absolute maximums, minimums, or saddle points over a given region.
- **Practice problems 3, 10, 22, 33, 49, 60**

15. a) Find the first and second order partial derivatives indicated for the following function: $f(x, y) = x^2 + x - 3xy + y^3 - 5$

$$f_x(x,y)$$

$$f_y(x,y)$$

$$f_{xx}(x,y)$$

$$f_{yy}(x,y)$$

$$f_{xy}(x,y)$$

Use second derivative test to find identify and classify critical points f or the above function.

16. The revenue function for a manufacturer is

$R(x, y) = -5x^2 - 8y^2 + 2xy + 140x + 440y$, where x is the number of units of tables produced and y is the number of chairs produced. Use the 2nd partial derivatives test to find the number of tables and number of chairs to produce to maximize the revenue, and the maximum amount of revenue.