M236 Calculus IV Review Guide First Test (Chapter 12 , 13.1 and 13.2)

Section 12.1: Vector-Valued Functions

- What is a vector-valued function; How to represent a vector valued function in plane and in space.
- Domain, Limits and Continuity of vector-valued functions. (Exercises #1, 2, 12)

1. Determine the domain of the vector-valued function $\mathbf{r}(t) = (2t^2 - 1)\mathbf{i} + \sqrt{1-t}\,\mathbf{j} + \frac{1}{t}\mathbf{k}$

2. Calculate $\lim_{t \to 1}(t\mathbf{i} + \frac{3t - 3}{t^2 - 1}\mathbf{j} + 2t^2\mathbf{k})$

3. Give the equation in rectangular coordinates for the plane curve represented by the vector valued function. Sketch its graph, showing its orientation
$\mathbf{r}(t) = 3t^2\mathbf{i} + (t+1)\mathbf{j}$

Section 12.2 Differentiation and Integration

- What is the derivative of a vector-valued function, How to compute the derivative of a vector-valued function. (Example 1, exercise 45)
- Determine whether a curve is smooth (Exercise 29, 32, 25)
- How to compute the indefinite and definite integral of a vector-valued function. (Examples 6, 7; exercise 50, 53)
- How to determine the antiderivative of a vector valued function that satisfies initial condition(s). (Like example 7 and exercise 67)

4. Given that $\mathbf{r}'(t) = t^2\mathbf{i} + t\mathbf{j} + e^{2t}\mathbf{k}$ and $\mathbf{r}(0) = -\mathbf{j} + \mathbf{k}$, find **r**(t).

5. Find the values of t, $0 \le t \le 2\pi$, for which the plane curve given by
$\mathbf{r}(t) = 2(t - \sin t)i + 2(1 - \cos(t))\,\mathbf{k}$ is smooth.

Section 12.3 Velocity and Acceleration

- Given a position vector function **r**(t), calculate velocity, acceleration, and speed. (like example 1)
- Given acceleration vector and initial conditions, find the position function (like example 4)
- Theorem 12.3: Position function for projectile motion with given initial launch angle, speed, and height. Know this formula and be able to derive it.
- Apply to problems like those assigned to find maximum height, horizontal distance traveled, etc. (Example 6, exercises 26, 28, 34)

6. Suppose that a ball is hit from a high of 4 feet above the ground with a launch angle of $\frac{\pi}{6}$ radians with initial speed of 64 feet per second. Assume an ideal projectile motion and gravity g is **32** ft. per second per second**.**
a) Calculate the time t when the ball hits the ground.
b) What is the total horizontal distance (range) that the ball travels?
c) Using the same height and launch angle what initial velocity must be used to achieve a horizontal distance of 100 feet?

Section 12.4: Tangent Vectors and Normal Vectors

- Definition of unit tangent vector.
- Definition of tangent line to a curve – be able to calculate from the position function (Example 2)

- Definition of principal unit normal vector.
- Given a position function **r**(t), be able to calculate unit tangent vector **T**(t), principal unit normal vector **N**(t), tangential and normal components of acceleration at a point. (Exercises 32 40, 42, 55)
- **Interpret** unit tangent vector, principal unit normal vector, and tangential and normal components of acceleration in terms of direction of motion and turning of object moving along the curve. (See also section 12.5, page 873 top for more.)

7. Calculate for the position function $\mathbf{r}(t) = 2\cos(t)\,\mathbf{i} - 2\sin(t)\,\mathbf{j} + t\,\mathbf{k}$

a) velocity vector **v**(t) b) speed at time t.
c) acceleration vector **a**(t) d) Unit tangent vector T(t) at time t = π/2
e) Principal unit normal vector N(t) time t = π/2 f) curvature K at time t = π/2
g) Tangential component of acceleration at t = π/2 h) Normal component of acceleration at time t = π/2
i) Parametric equations of the line at time t = π/2
j) Give the decomposition of acceleration as a linear combination of T and N.

8. a) Interpret the unit tangent vector for a curve representing a particle in motion.
b) Interpret the principal unit normal vector N for a curve.

9. Interpret the tangential and normal components of acceleration: what does each tell us about acceleration?

Section 12.5: **Arc Length and Curvature**

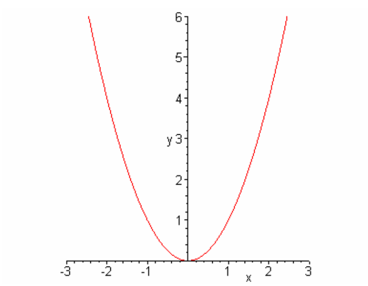
- Define informally and formally the arc length of a curve; Calculate the arc length of a curve r(t). (Exercise 10, 11)
- Define the arc length parameter s; Explain ds/dt formula by using 2^{nd} Fund. Theorem of Calc.
- Define and interpret circle of curvature and radius of curvature. (See lab)
- Calculate the curvature K of a curve. (Exercises 33, 38)

10. Set up and evaluate the integral representing the arc length of the curve C given by $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + \ln(t)\mathbf{k}, \quad 1 \le t \le 2$

11. Describe in precise terms what the curvature K of a curve measures, in terms of its formal definition.

12. a) Write down the definition of the arc length parameter s(t) and explain why the derivative ds/dt is the magnitude of the velocity vector using the 2^{nd} Fundamental Theorem of Calculus.
b) Using the Chain Rule and the above result derive the formula for curvature

13.



At left is the curve $y = x^2$
a) Compute the curvature K for this curve at the point (0,0). Show work.
b) What is the radius of the circle of curvature at this point?
c) ***Sketch the circle of curvature at (0,0) on the above graph.***

Chapter 13 Functions of Several Variables
Section 13.1

- Understand concept of a function of several variables Evaluate a function of several variables at given value(s). (Exercise 5)
- Determine domain of a given function of several variables. (Exercise 22)
- Know definition of level curves for a function f(x,y) – describe for a given function (Exercise 51, 56)

14. a) Find the domain of the function $f(x,y)=\sqrt{32-4x^2-y^2}$

 b) Evaluate $f(1, 2)$

15. Describe the level curves of the surface represented by the function $f(x,y)=\ln(y-x^2)$.

Section 13.2 Limits and Continuity

- Understand definitions and interpretations of limits and continuity applied to functions of several variables.
- Determine the limit of a function at a point.
- Determine whether a function is continuous at a point.
- Apply to problems (Exercises: 9, 13, 20, 26)

16. Show that the following limit does not exist. Give a clearly written explanation.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$