Mth235 Calc III Review Guide for Test 3

Section 10.4 Polar Coord. and Section 10.5 polar areas, arc length, area of surface of rev; Chapter 11: Vectors Sections 11.1, 11.2, 11.3, 11.4, (11.5 -- maybe)

Section 10.4
Given the polar coordinates, calculate the rectangular coordinates of a point (Showing work)
Example 1, Exercise 1 and 4
Given the rectangular coordinates, calculate the polar coordinates of a point (Showing work).
Plot point using polar coordinates.
Example 2, Exercises 12, 13
Given a rectangular equation of a curve, convert to polar and vice-versa.
Exercises 27, 28, 30, 34, 35, 38, 40, 41
Convert curve given in polar coordinates to parametric form, and use to calculate derivative, slope, tangent lines, identify horizontal and vertical tangents (as done in section 10.3)
Example 6, exercise 70, 73

Section 10.5
Find the area of a polar region. Find the points of intersection of two polar graphs. Find the area between two polar curves. Find the arc length of a polar curve. Find the area of a surface of revolution for a polar curve.
Examples 1,2, 4 and exercises 5, 6, 12, 13, 51, 70

Section 11.1 and 11.2

Representation of a vector in its component form as well as in terms of the standard unit vectors i, j, k
Calculate c**v**, **v+w**, **v-w both algebraically and geometrically --be able to draw.**
What is means for two vectors to be parallel.
Length (magnitude) of a vector -- calculate
Zero vector, unit vector.
Given **v,** calculate a unit vector in the same direction and calculate a vector of length c in the same (or opposite) direction.
Standard unit vectors **i, j, k**
Right-handed vs. left-handed coordinate systems.
Calculate vector between two points.
Calculate distance between two points.
Recognize equation of a sphere, calculate center, radius of sphere given equation or vice-versa.
Compute the resultant force -- like problems 82, 83, section 11.1

Section 11.3 Dot Product
Given two vectors, calculate their dot product.
Calculate angle (or cosine of angle) between two vectors.
Show two vectors are orthogonal using their dot product.
Recognize properties of dot products (Theorem 11.4 -- T -F questions)
Given two vectors u and v, a) calculate the projection of u onto v. b) Write u as the sum of orthogonal components using the projection. (Like example 5).
Compute work done by force F moving object in direction of vector PQ. (Like Example 7)
Use the projection vector and orthogonal decomposition to discuss and solve application problems like Example 6.

Section 11.4 Cross Product
Don't have to memorize formula for -- I will give you the formula.
Geometric interpretation of **u X v** and **v X u**
Calculate the area of a triangle or parallelogram with sides **u** and **v**.
Use the cross product to determine whether u and v are parallel, orthogonal, or neither.

Section 11.5 Lines and Planes in Space)
Given a point and a vector in space, find the parametric equation of the line through the point and parallel to the vector.
Given two points in space, find the parametric equation of the line through those two points.
Find the equation of a plane with normal n and through point P.
Given three points in a plane, a) Calculate a normal to the plane. b) Calculate the equation of the plane.
Given the equation of two planes, determine whether they are parallel, determine the angle of intersection, determine the equation of the line of intersection of the two planes.
Recognize the equation of a plane parallel to any of the coordinate planes or parallel to any coordinate axis.

Calc III Practice Questions for Exam 3

1 a) Find the rectangular coordinates for the polar point (-4, π/6).
b) Find two pairs of polar coordinates for the point (-1,-1) -- one with a positive r and one with a negative r.

2. Convert the rectangular equation $x^2 + y^2 - 2y = 0$ to polar form.

3. a) Convert the following polar equation to parametric form:

$$r = 1 - \cos\theta, \quad 0 < \theta < 2\pi$$

b) Find slopes of the curve and equation of the tangent line at the point (1,π/2)

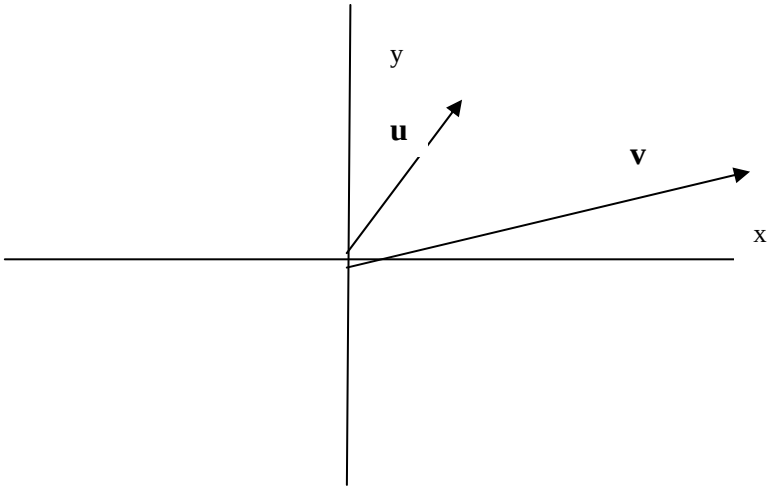
4. Graph the two polar cuves r = 2 sin θ and r = 2cos θ and then find

a) **all** points of intersection . At (r, θ) = $(\sqrt{2}, \frac{\pi}{4})$ At pole: On curves r = 2 sin θ at

b) the area of the common interior of r = 2 sin θ and r = 2cos θ

5. Set up the integrals to :
a) Find the arc length of the polar curve of $r = 1+\cos(\theta)$ between 0 and 2π.
b) Find the surface area of the solid of revolution obtained by rotating $r = 1+\cos(\theta)$ about the polar axis.

6. Draw and label on the graph below the vectors a) -2**u** b) **v - u** c) **v + u**



7. Given the vectors **u** = < 1, 2, -1> and **v** = <3, -1, 4> , find
a) the length of **u**
b) **u**·**v**, the dot product of **u** and **v.**
c) a unit vector in the same direction as **u**
d) vector of length 2 in the same direction as **u**
e) the cosine of the angle between **u** and **v**
f) $\text{proj}_{\mathbf{v}}\,\mathbf{u}$, the projection of **u** onto **v**
g) the vector **u** – **v**
h) write **u** in terms of the standard unit vectors **i**, **j**, and **k**:

8. Which of the following are NOT a properties of the dot product
a) $\mathbf{u}\cdot\mathbf{v}=\mathbf{v}\cdot\mathbf{u}$ b) $\mathbf{u}\cdot c\mathbf{v}=c\mathbf{v}\cdot\mathbf{u}$ c) $\mathbf{u}\cdot(\mathbf{v}+\mathbf{w})=\mathbf{v}\cdot\mathbf{u}+\mathbf{v}\cdot\mathbf{w}$ d) $\mathbf{u}\cdot(\mathbf{v}\cdot\mathbf{w})=(\mathbf{u}\cdot\mathbf{v})\cdot\mathbf{w}$

T F 9. **u** x **v** = **v** x **u**
T F 10. **v** x **u** is orthogonal both to **u** and **v**.
T F 11. **u** x **v** = **0 (zero vector)** if and only if **u** and **v** are parallel.
T F 12. **u**·**v =** 0 if and only if **u** and **v** are orthogonal.

13. Find the work done by applying a force of 25 pounds at an angle of 60 degrees with the horizontal to push an object 50 feet along a horizontal floor

14. Suppose we have a 20,000 pound vehicle parked on a ramp with an incline of 10 degrees. Find the magnitude of the force required to keep the vehicle from rolling off the ramp, and the magnitude of the force perpendicular to the ramp (i.e. pushing against the ramp). ***Draw a sketch*** representing the forces as vectors and projection vectors. Justify your work clearly using complete sentences.

15. Which of the following represents the area of the parallelogram with sides **AB** and **AC**
a) $|\mathbf{AB}\cdot\mathbf{AC}|$ b) $\|\mathbf{AB}\times\mathbf{AC}\|$ c) $|\mathbf{AB}\cdot(\mathbf{AC}\times\mathbf{BC})|$ d) $\|\mathbf{AB}\|\,\|\mathbf{AC}\times\mathbf{BC}\|$

16. Compute the area of the triangle and parallelogram with the vertices (1,3,5), (3,3,0), and (-2,0,5).

17. Given the three points P = (1, 2, -3), Q = (2, 3, 1), and R = (0, -2, -1).
a) Find the vector **PQ** b) Find the vector **PR**
c) Find the vector **n** = **PQ** X **PR** d) Give the equation of plane through points P, Q, R

18. Write down the equation of the plane passing through the point (1,3,0) and having normal N = <1,-1,1>

19. Find the angle of intersection between the two planes 3x +2y -4 = 0 and 5x + 2y -z = 2 or if the planes parallel, so indicate.

20. Find the parametric equations for the line of intersection of the two planes
$3x + 2y - z = 7$ and $2x - 8y - 4z = 0$.

21. Describe the following planes in space:
$3z = -5$
$x + 2y = 7$