

M235 Review Guide for Second Test over Covered Parts of Chapters 9 and Chapters 10
Test Date: Friday October 23rd

9.8 Power Series

Recognize a power series

For a given power series, use the tests for convergence from earlier sections to determine the radius and interval of convergence. (problems 11, 14, 17, 25)

9.9 Representation of Functions by Power Series

Be able to use the geometric series formula to find a power series for certain functions (like example 1, 2, exercise 2, 3, 5, 9)

Given a power series for a function, use integration, differentiation to find power series for the antiderivative or derivative of that function -- like examples 4 and 5 and exercises 19, 20, 21.

Section 9.10 Taylor and Maclaurin Series

Calculate the Taylor Series for a simple function from its definition (like example 1, exercises 2, 7)

Given a Taylor Series for a function, substitution or other algebraic manipulation to find a new Taylor Series for a different function (like example 3, exercises 21, 24)

Also be able to approximate a definite integral by the partial sum of a power series within a desired error (like example 8 and exercise 55, 58.)

Section 10.1: Recognize geometric definitions for ellipse, circle, hyperbola, parabola. I will give you the statements of the theorems for each of the standard equations. Given an equation, identify it and put into standard form, then identify things such as focus, vertex, center, as appropriate to equation.

Problems like 13, 14, 22, 25, 34, 41, 44, 48, 51, 62, 68, 69, 71, 73.

Section 10.2

Given the (simple) parametric equation of a curve:

Eliminate the parameter to get the rectangular equation (with appropriate domain for x) – like examples 2 and 3. For simple curves sketch and show orientation.

Problems like 5, 10, 15, 19, 20

Section 10.3

Given a parametric equation, know how to calculate the derivative dy/dx and use it to calculate the slope of a tangent line (and thus equation of the tangent line) to the curve at a given point and identify any horizontal and vertical tangents (like exercises and Examples 2 and 3). Also give equation of the tangent line to the curve at a given point.

Calculate the arc length of a simple parametric curve (using formula from Theorem 10.8) (Example 4, Exercise 48, 49)

Calculate the area of the surface of revolution of a curve about an axis. (Example 6, Exercise 68, 69)

Calc III Practice Questions for Exam 2

1. Which is the geometric power series centered at 0 for the function

$$f(x) = \frac{4}{1-x^2}$$

a) $\sum_{n=0}^{\infty} 4(-x^2)^n$ b) $\sum_{n=0}^{\infty} 4(1-x^2)^n$ c) $\sum_{n=0}^{\infty} 4x^n$ d) $\sum_{n=0}^{\infty} 4x^{2n}$

2. For the following power series, determine the radius and the interval of convergence. Show your work in detail.

a) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^n}$

3. a) Write down a power series centered at 0 for $\frac{1}{1-x}$
 b) Use it to find the power series for $\frac{1}{(1-x)^2}$. (Use differentiation)
 c) Identify the interval of convergence for each.

4. Write down the geometric power series centered at 0 for the function

$$f(x) = \frac{2}{1-3x}$$

5. What is the correct formula for the general Taylor *series* for the function $f(x)$ centered at c ?

6. Given the power series expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

a) Write down the power series for

$$e^{-x^2}$$

b) Write down the power series for the indefinite integral

$$\int e^{-x^2} dx$$

c) Use part b) to calculate an approximation for the integral $\int_{x=0}^{0.5} e^{-x^2} dx$

that has error less than .001. Show all work.

7. a) The following is an equation of (circle correct): ellipse circle parabola hyperbola

$$y^2 - 2x - 6y + 11 = 0$$

b) Put the above equation into standard form and write down the vertex, focus and directrix.

8. Set of all points (x,y) for which the difference of the distances from two fixed points is constant.

Circle correct: ellipse circle parabola hyperbola

9. a) Eliminate the parameter and find a corresponding rectangular equation for the parametric equation:

$$x = 3\sqrt{t}$$

$$y = 2t + 1$$

b) Specify the domain for x .

c) Sketch the graph and show the orientation

10. Determine the rectangular equations for the curve and then sketch the curve represented by the parametric equations

$$x = 4 \sin \theta$$

$$y = 3 \cos \theta$$

11. a) Find the equation of the tangent line for the parametric curve $x = 2t$ and $y = t^2 + 5$ at the point where $t = 1$.

b) Identify any points of horizontal or vertical tangency for this curve

12. Find the arc length of the curve given by $x = t^2$ and $y = 2t^2 - 1$ for $1 < t < 4$.

13. Find the surface area of the surface of revolution formed by revolving the curve

$$x = 4 \sin \theta$$

$$y = 4 \cos \theta, \quad 0 < \theta < \frac{\pi}{2}$$

about the x axis.