

## Review Guide M122 3<sup>rd</sup> Exam Wednesday, April 15th

### Section 4.1 Mathematical Induction

Mathematical induction: know method and terms:

Principle of Mathematical Induction, basis step, inductive step, induction hypothesis.

**You will have to do an inductive proof.** It will one that you have seen before or will be very similar to one that you have seen before.

1. a) State the rule of inference that is known as the Principle of Mathematical Induction.  
b) In this rule proving  $P(1)$  is called the \_\_\_\_\_  
c) Proving  $P(k) \rightarrow P(k+1)$  is called the \_\_\_\_\_  
d)  $P(k)$  is called the \_\_\_\_\_.
2. Explain the relationship between mathematical induction and dominoes.
3. Use mathematical induction to prove that  
$$2^0 + 2^1 + \dots + 2^n = 2^{n+1} - 1$$

### Section 7.1 Recurrence Relations

What is a recurrence relation.

Given a recurrence relation for a sequence, write down the first several elements in the sequence.

Given a recurrence relation, show whether a given sequence satisfies the recurrence relation.

Write down the recurrence relation for Fibonacci numbers and for factorial sequence.

4. a) Write down the recurrence relation for the Fibonacci numbers.  
b) Calculate the first 6 Fibonacci numbers.

5. For the recurrence relation  $a_n = 2a_{n-1} - a_{n-2}$ , where  
 $a_1 = 4, a_2 = 8$

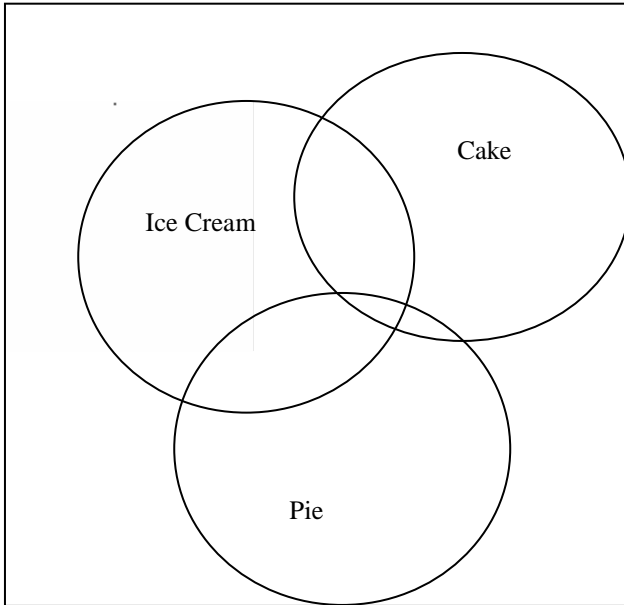
- a) Evaluate  $a_3$  \_\_\_\_\_  $a_4$  \_\_\_\_\_
  - b) Show that  $a_n = 4n$  is equivalent to the above definition.
6. a) Give the recurrence relation for  $a_n = n!$   
b) Give the recurrence relation for the Fibonacci numbers  
c) Give a recurrence relation for  $a_n = 3^n$

### Section 7.5 Equivalence Relations

Apply the principle of inclusion to answer questions involving as many as three sets like assigned problems and ones done in class. Fill in associated Venn Diagram.

7. In a dinner for 200 students for dessert, there were 60 ice creams served, 80 pieces of cake, 70 pieces of pie. Some students were really hungry so 30 students ate both ice cream and cake, 25 ate both cake and pie, and 20 ate both ice cream and pie, and amongst these, 15 students in fact ate all three – cake, pie, and ice cream.

a) In the Venn Diagram representing this, **label all regions** with the appropriate numbers.



**Section 9.1 and 9.2**

Graph Terminology

Simple, directed, pseudo, multigraph, vertices, edges, degree, Handshaking Theorem.

Special types of graphs: Connected, cycle, wheel, hypercube, bipartite.

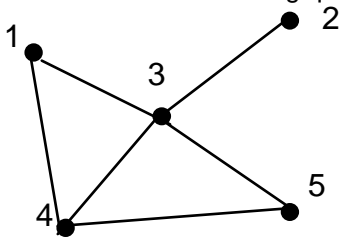
Give several examples of applications of graphs.

Subgraph

Union of two graphs.

- 8. Draw a **complete** graph with four vertices.
- 9. Draw a wheel with five vertices.
- 10 Draw a cycle with six vertices.
  
- 11. Draw a complete bipartite graph with 2 vertices in one set and 4 vertices in the second set.

12. Answer the following questions for the graph below:



- a) Identify it as simple, pseudo graph, multigraph, directed graph.
- b) What is the degree of vertex 3.
- c) Is the graph bipartite? Justify your answer.

**Section 9.3**

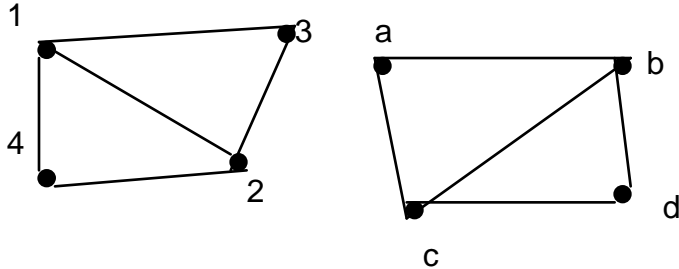
Adjacency List, Adjacency Matrix representations for graphs

Graph isomorphism -- determine whether two graphs are isomorphic. If they are, write down the isomorphism.

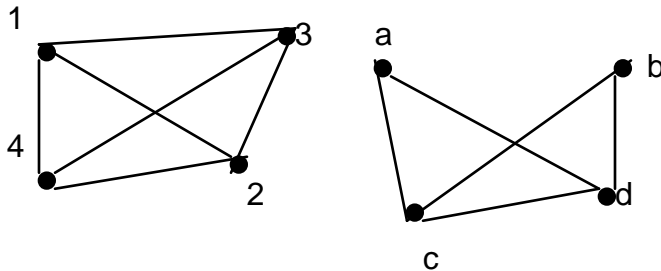
13. For the graph in question 12

- Write down the adjacency list representation for the graph.
- Write down the adjacency matrix representation for the graph.

14. a) The following two graphs **are** isomorphic. Show this by writing down an isomorphism for the two graphs.



b) The following two graphs **are not isomorphic**. Explain why they cannot be isomorphic.



### Section 9.4

Paths, path length, circuit, simple circuit

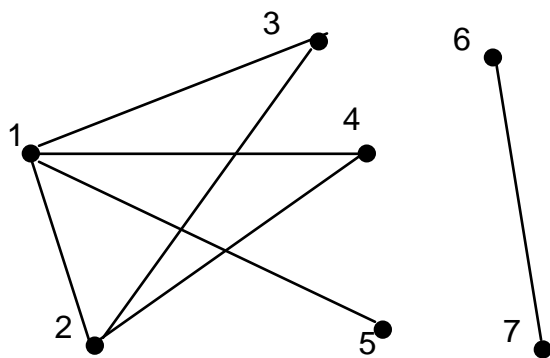
For a graph, identify its connected components.

For a directed graph identify its strongly and weakly connected components.

Terms cut vertex, cut edges -- identify in a given graph.

15. For the graph in question 12:

- Write down a simple circuit of length 3 in this graph.
- Is it connected?
- Identify any cut vertices
- Identify any cut edges.



16. a) Circle in the graph at the left the connected components.

b) which of the following are valid subgraphs -- explain of the graph on the left.

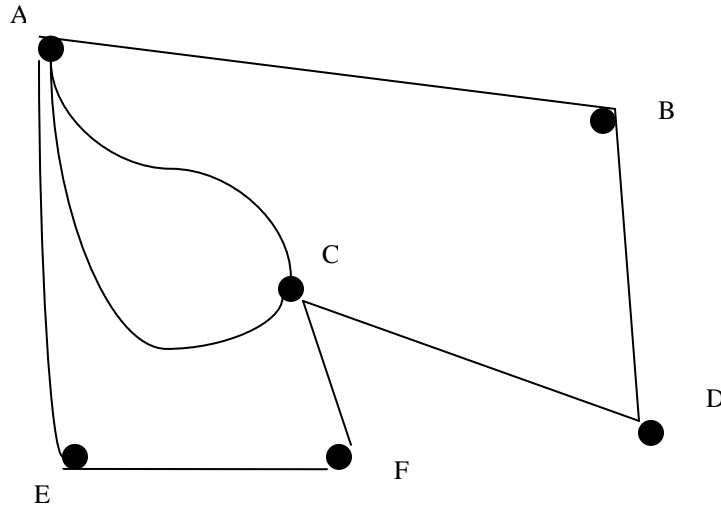
i)  $(V_1, E_1)$ , where  $V_1 = \{1, 2, 4, 6\}$ ,  $E_1 = \{(1,4), (2,4)\}$

ii)  $(V_2, E_2)$ , where  $V_2 = \{1, 3, 4\}$ ,  $E_2 = \{(1,3), (1,4), (2,4)\}$

### Section 9.5

Euler path, Euler circuit, Hamiltonian path, Hamiltonian circuit. How to determine whether a given graph has any of these. Problems as on the worksheet and assigned problems.

17. a) Determine whether the following has an Euler circuit, or an Euler path which is not a circuit, or neither. Justify your answer.



b) Determine whether the above graph has a Hamilton circuit – If it does indicate the path (list vertices in order visited). If not explain why.

### Section 8.1 Relations and Their Properties

Definition of binary relation from a set A to a set B; binary relation on a set A  
For a given relation, determine whether it is reflexive, transitive, symmetric, antisymmetric, irreflexive -- justify your answers.

18. For the relation “Is taller than” on the set of people, which properties does it have: reflexive, transitive, symmetric, antisymmetric, irreflexive -- justify.

### Section 8.3 Representing Relations

Know how to represent a relation using an adjacency matrix; using a directed graph.

Be able to use these representations to determine reflexive, transitive, symmetric, antisymmetric.

19. Draw the **directed** graph for the relation on the set of integers {1,2,3,4} represented by the adjacency matrix:

$$\begin{array}{c}
 1 \quad 2 \quad 3 \quad 4 \\
 \begin{pmatrix}
 0 & 1 & 0 & 1 \\
 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 \\
 1 & 0 & 1 & 0
 \end{pmatrix}
 \end{array}$$

b) Which properties does this relation have?  
symmetric, transitive, reflexive, irreflexive, antisymmetric (circle).