Discrete Mathematics Review Guide for First Test Friday Feb. 3rd

Section 1.1 Logic
Term: proposition – recognize whether a statement is a proposition.
Logical operators and their symbols.
Be able to evaluate bit (logical) operations on bit strings
Given an implication as an English sentence, states its converse, inverse, and contrapositive as English sentences.
Section 1.3 Propositional Equivalences
Be able to fill in truth tables for compound propositions.
Be able to show two compound propositions are logically equivalent using truth tables
Know what a tautology, contingency, contradiction are, and be able to show a proposition is a tautology using truth tables
Section 1.4 Predicates and Quantifiers
Know what the two quantifiers $\forall \text{ and } \exists$ (universal and existential) mean.
Know how to negate quantifiers.
Be able to translate SIMPLE sentences to logical expressions and vice versa
Section 1.6 Rules of Inference
Recognize fallacy of affirming conlusion and fallacy of denying the hypothesis.
I will give you the two tables with the rules of inference (Table 1), rules of inference for Quantified Statements (Table 2.)
Write a given a rule of inference as an equivalent tautology.
Given an argument, list the reasons (rules of inference and reasons used for each statement in the argument). Fill in missing parts (steps and reasons) of an argument.
Section 2.1 Sets
What a set is, set builder notation, subset, cardinality of a set, null (empty set) power set of a set. Cartesian product of two sets.
Section 2.2 Set Operations
Know set operations and notation for them: union, intersection, complement, difference.
Given two (or more sets), perform set operations on them, writing down resulting set.
Draw a Venn diagram, shading a particular compound set.
Verify set identities using set membership tables.
Section 2.3 Functions
Terms: function, domain, codomain, range, image, one-to-one function, onto function.
When does inverse of function exist?
For a given function , is it one-to-one, onto, invertible? Find its inverse (if it exists).
Floor function, ceiling function.

Practice Questions from Old Tests

1. Let **p** be the proposition "It is Wednesday". Let **q** be the proposition "Paul plays golf".

a) Write down the English sentence which is the equivalent of **p → q**

b) Write down in English the ***converse*** of the proposition in part a).

c) Write down in English the ***contrapositive*** of the proposition in part a).

d) Write down in English the ***inverse*** of the proposition in part a).

2. A proposition that is **always false** is called a

a) tautology b) contingency c) fallacy d) contradiction

3. Calculate the following bit operations (Here ⊕ denotes the exclusive or (XOR)).

$1001 \wedge 1010$ ______________ $1001 \oplus 1010$ ______________

4. Are the following two compound propositions are ***logically equivalent***? _________

Verify with truth tables.

$\neg(p \wedge q)$ and $(\neg p) \vee (\neg q)$

5. Determine whether the following implication is a ***tautology*** by using truth tables:

$\left[(p \rightarrow q) \wedge \neg q\right] \rightarrow \neg p$

7. Let the universe for discourse of *x* be the set of all Discrete Math students. Let D(x) be the propositional function " *x has a dog*." notation with quantifiers.

a) Translate the following using propositional

Every student in Discrete Mathematics has a dog.

b) Give the negation of the statement in a) first in propositional notation, then in English.

8. Let the universe of discourse for x be students at SJC. Let M(x) = "x must take a math course." The correct translation into propositional notation of "Every student at SJC must take a math course" would be:

a) $\exists x\, M(x)$ b) $\forall x M(x)$ c) $M(SJC)$ d) $M(\exists x, SJC)$

Now write down the negation of the above statement in both logical notation and English:

9. Which of the following is a correct **negation** of the quantification $\exists x\ P(x)$?

a) $\forall\, x\, \neg P(x)$ b) $\exists\, x\, \neg P(x)$ c) $\forall\, x\ P(\neg x)$ d) $\neg \exists\, x\ P(x)$

10. Which rule of inference is used in each of the following (See attached sheet).

______________________________ a) Math is easy and English is hard. Therefore English is hard.

______________________________ b) If I stay up late, then I will be tired. If I am tired, I will not do well on the test. Therefore if I stay up late, I will not do well on the test.

_____________________________ c) You are either going to the movies or you are studying. You are not studying. Therefore you are going to the movies.

11. *If you have been drinking, then your eyes are red. You have not been drinking. Therefore your eyes are not red.*

Which one of the following applies:

a) A valid argument, using modus ponens.

b) A valid argument, using modus tollens.

c) An invalid argument, using the fallacy of denying the hypothesis.

d) An invalid argument, using the fallacy of affirming the conclusion.

12. Complete the missing parts of the proof of the following:

Premises: If Jim plays golf every day, he will improve his game.
If Jim improves his game, he will lower his handicap.
Jim does not lower his handicap.

Conclusion: Jim does not play golf every day

Let G = "Jim plays golf every day."
Let I = "Jim will improve his game. " Let L = "Jim lowers his handicap".
Use the attached table of rules of inference for reference.

Statement	Reason
1. $G \rightarrow I$	1.
2. $I \rightarrow L$	2.
3. $\neg L$	3.
4. $\neg I$	4.
5.	5.

13. Complete the missing parts of the proof of the following:

Premises: Every car that is a Jaguar is fun to drive

My car is not fun to drive

Conclusion: Therefore my car is not a Jaguar.

Let the domain for x be Cars.

Let J(x) be x is a Jaguar. Let F(x) be "x is fun to drive". Use the attached table of rules of inference for reference.

Statement	Reason
1. $\forall x\ J(x) \to F(x)$	1.
2. $J(my\ car) \to F(\ my\ car)$	2.
3.	3.
4.	4.

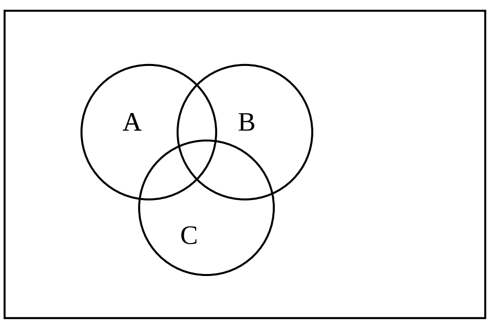
16. Write down the cardinality of each of the following sets:

a) { 1, 3, 8 } ________ b) { 1, { 2, 3}, {4,{5}} {6, 7}} ________

17. Assume the universal set U = {1,2,3,4,5,6,7,8,9,10}. Let B = {2,4,6,8} and C = {1, 2, 3, 4, 5}. Find each of the following sets:

a) $B \cap C$ b) $B \cup C$ c) $\overline{B}$ d) $B - C$

18. Shade in the Venn Diagram the set $\overline{A} \cap (B \cup C)$.



19. Let A = {1, 2}, B = {3, 4, 5}.

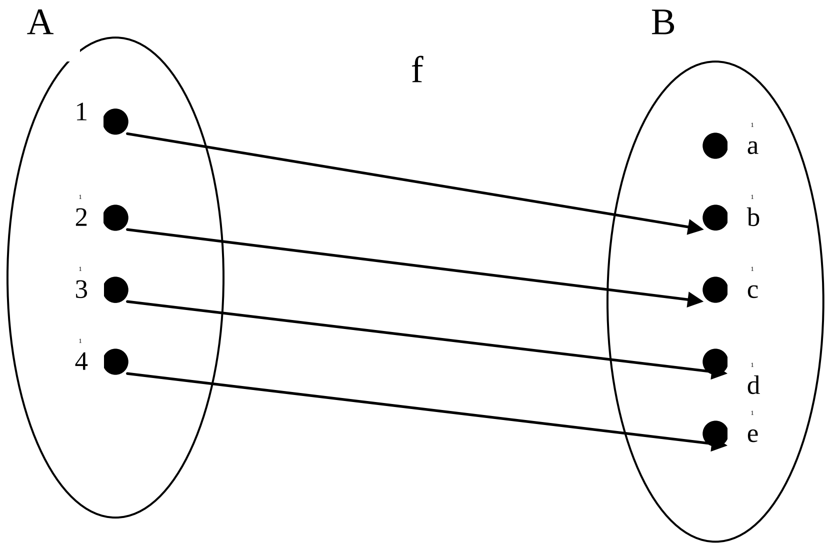
Write down the set representing the Cartesian product of A and B, $A \times B$

20. Use a membership table to show that the following two sets are equal:

$\overline{A \cap B}$ and $\overline{A} \cup \overline{B}$

Section 2.4 Functions

21. Consider the following representation for a function f: A → B



a) This function is onto (TRUE , FALSE)
b) This function is one-to-one. (TRUE, FALSE)
c) The inverse of this function does not exist (TRUE, FALSE)
d) The domain of this function *f* is the set ___________________
e) The range of this function *f* is the set _______________
f) The image of **4** is _____________

22. Let f be the function that assigns to a bit string the number of bits in the string.
a) What is the domain of this function?
b) What is its range?
c) Is it one-to-one?
d) Is it invertible?

23. Evaluate the following:
$\lfloor 3.5 \rfloor$ $\lfloor -3.5 \rfloor$ $\lceil 7.8 \rceil$ $\lceil -7.8 \rceil$