MTH 122 Discrete Mathematics Practice Questions from Old Tests

1. Let **p** be the proposition "It is Wednesday". Let **q** be the proposition "Paul plays golf".
a) Write down the English sentence which is the equivalent of **p → q**
b) Write down in English the ***converse*** of the proposition in part a).
c) Write down in English the ***contrapositive*** of the proposition in part a).
d) Write down in English the ***inverse*** of the proposition in part a).

2. A proposition that is **always false** is called a
a) tautology b) contingency c) fallacy d) propositional certainty
e) contradiction

4. Calculate the following bit operations (Here ⊕ denotes the exclusive or (XOR)).
$1001 \wedge 1010$ ______________ $1001 \oplus 1010$ ______________

5. Are the following two compound propositions are ***logically equivalent***? __________
Verify with truth tables.

$\neg(p \wedge q)$ and $(\neg p) \vee (\neg q)$

6. Determine whether the following implication is a ***tautology*** by using truth tables:

7. Let the universe for discourse of *x* be the set of all Discrete Math students. Let D(x) be the propositional function " *x has a dog*." notation with quantifiers.
a) Translate the following using propositional
Every student in Discrete Mathematics has a dog.
b) Give the negation of the statement in a) first in propositional notation, then in English.

8. Let the universe of discourse for x be students at SJC. Let M(x) = "x must take a math course." The correct translation into propositional notation of "Every student at SJC must take a math course" would be:
a) $\exists x\, M(x)$ b) $\forall x M(x)$ c) $M(SJC)$ d) $M(\exists x, SJC)$

Write down the negation of the above statement in both logical notation and English:

9. Which of the following is a correct **negation** of the quantification $\exists x\ P(x)$?
a) $\forall\, x\, \neg P(x)$ b) $\exists\, x\, \neg P(x)$ c) $\forall\, x\ P(\neg x)$ d) $\neg\exists\, x\, P(x)$

10. Which rule of inference is used in each of the following (See attached sheet).

______________________________ a) Math is easy and English is hard. Therefore English is hard.

______________________________ b) If I stay up late, then I will be tired. If I am tired, then I will not do well on the test. Therefore if I stay up late, I will not do well on the test.

_____________________________ c) You are either going to the movies or you are studying. You are not studying. Therefore you are going to the movies.

_____________________________ d) All good men come to the aid of their country. Sal did not come to the aid of his country. Therefore Sal is not a good man.

11. *If you have been drinking, then your eyes are red. You have not been drinking. Therefore your eyes are not red.*
Which one of the following applies:
a) This is a valid argument, using modus ponens.
b) This is a valid argument, using modus tollens.
c) This is an invalid argument, using the fallacy of denying the hypothesis.
d) This is an invalid argument, using the fallacy of affirming the conclusion.

12. Complete the missing parts of the proof of the following:
Hypotheses:
If Jim plays golf every day, he will improve his game.
If Jim improves his game, he will lower his handicap.
Jim does not lower his handicap.
Conclusion:
Jim does not play golf every day

Let G = "Jim plays golf every day."
Let I = "Jim will improve his game. " Let L = "Jim lowers his handicap".

Use the attached table of rules of inverence for reference.

Statement	Reason
1. $G \to I$	1.
2. $I \to L$	2.
3. $\neg L$	3.
4. $\neg I$	4.
5.	5.

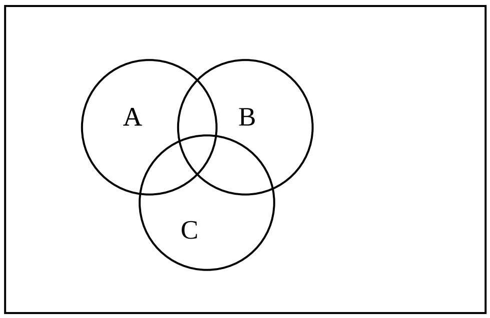
13. Proving that the implication $\mathbf{p} \to \mathbf{q}$ is true by showing $\neg q \to \neg p$ is an example of
a) proof by contradiction b) proof by contraposition
c) proof by counter example d) direct proof.

14. Give a direct proof of "If n is even then n^2 is even".

15. Write down the cardinality of each of the following sets:
a) { 1, 3, 8 } ________ b) { 1, { 2, 3}, {4,{5}} {6, 7}} ________

16. Assume the universal set U = {1,2,3,4,5,6,7,8,9,10}. Let B = {2,4,6,8} and C = {1, 2, 3, 4, 5}. Find each of the following sets:
a) $B \cap C$ b) $B \cup C$ c) $\overline{B}$ d) $B - C$

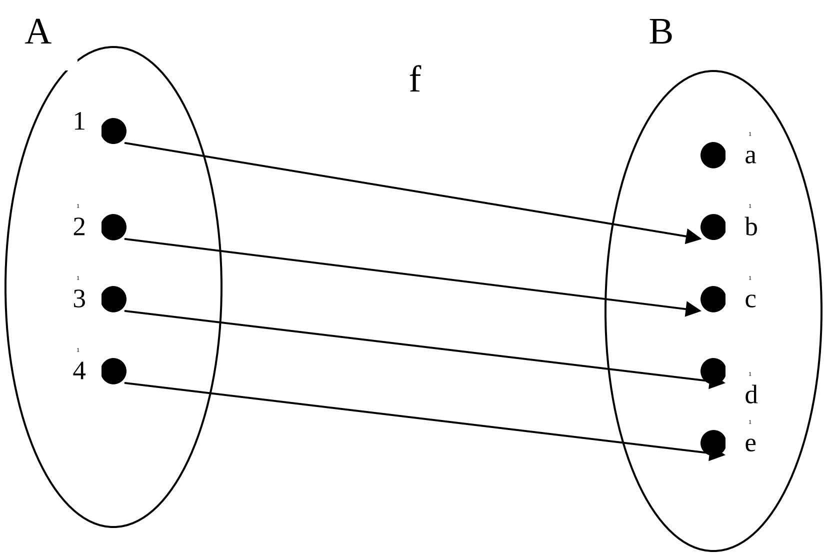
17. Shade in the Venn Diagram the set $\overline{A} \cap (B \cup C)$.



18. Let A = {1, 2}, B = {3, 4, 5}.
Write down the set representing the Cartesian product of A and B, $A \times B$

19. Use a membership table to show that the following two sets are equal:
$\overline{A \cap B}$ and $\overline{A} \cup \overline{B}$

20. Consider the following representation for a function f: $A \rightarrow B$



a) This function is onto (TRUE , FALSE)
b) This function is one-to-one. (TRUE, FALSE)
c) The inverse of this function does not exist (TRUE, FALSE)
d) The domain of this function *f* is the set ___________________
e) The range of this function *f* is the set _______________
f) The image of **4** is _____________

21. Let f be the function that assigns to a bit string the number of bits in the string.
a) What is the domain of this function?
b) What is its range?
c) Is it one-to-one?
d) Is it invertible?

22. Calculate the composition $f \circ g\ (x)$ where $f(x) = x^2$ and $g(x) = 2x - 1$

23. Assume that data is transmitted at the rate of 400 kilobits per second, where 1 kilobit = 1000 bits, and 8 bits = 1 bytes. Assume that an ATM cell contains 53 bytes. Which of the following expressions represents how many ATM cells can be transmitted in 5 seconds?

a) $\left\lfloor \frac{5 \cdot 400 \cdot 8 \cdot 1000}{53} \right\rfloor$ b) $\left\lfloor \frac{5 \cdot 400 \cdot 1000}{8 \cdot 53} \right\rfloor$ c) $\left\lceil \frac{400 \cdot 1000}{5 \cdot 8 \cdot 53} \right\rceil$ d) $\left\lceil \frac{400 \cdot 1000}{5 \cdot 8 \cdot 53} \right\rceil$

24. Let { a_n } be the sequence defined by $a_n = 2n^2$. Then a_3 =_____________

25. If tuition at SJC starts at $10,000 and increases by the fixed amount of $500 per year, the sequence yearly tuition costs is represented by (Circle which)
a) The geometric progression $10{,}000(500)^n$

b) The arithmetic progression 10,000 + 500n

26. Compute the following summations:

a) $\sum_{j=0}^{8} 3(2^j)$ b) Evaluate $\sum_{i=1}^{3} \sum_{j=1}^{2} (ij + 1)$