Discrete Mathematics Class Discussion Notes

Section 6.1 Discrete Probability

Fundamentals of Probability

- **Experiment**: Procedure that yields one of a given set of possible outcomes.
  - Example: Roll a die, observe what number comes up.
- **Sample space**: Set of all possible outcomes. Also called outcome space. For above example, the sample space is {1, 2, 3, 4, 5, 6}.
- **Event:** A subset of the sample space.
  - Example: In the die experiment, rolling an even number --the set {2, 4, 6}
- **Definition 1:** The **probability** of an event E, which is a subset of a finite sample space S of *equally likely outcomes*, is defined to be *p*(E) = |E|/|S| -- i.e. the number of elements in E divided by the number of elements in the sample space S.
- **Example 2:** Probability of rolling a sum of 7 when two dice are rolled.
- **Example 3**: Lottery: Win by matching four digits exactly. Probability of winning is 1/10^4
- **Example 4:** Lottery: Win by matching 6 numbers out of 40. 1/C(40,6)
- **Example 6**: Probability that a hand of five cards in poker contains four cards of one kind.
- **Example 7**: Probability that a hand contains a full house (3 of one kind, 2 of another).
- **Theorem 1:** Probability of the complement of an event E , is given by 1 - P(E).
  - Often used to calculate the probability in "At least" type problems.
  - **Example 8**: What is the probability in generating 10 random bits, that at least one is a zero bit.
- **Theorem 2**: Let E1 and E2 be events in the sample space S. Then

  p(E1 union E2) = p(E1) + p(E2) - p(E1 intersect E2).
  - This follows form the principle of inclusion and exclusion for sets.
- **Example 9**: Probability of a positive integer between 1 and 100 being selected at random that is divisible by either 2 or 5.

Section 6.2 (Parts of) Conditional Probability and Independent Events and the Binomial Distribution

- **Definition: Conditional probability.** The conditional probability of event E given F, is given by

  P(E|F) = P(E intersect F) / P(F).
- **Example** Toss a coin three times**.** Let F be the event "Get at a tail on the first toss". Let E be the event "Get an odd number of tails".

  The sample space S has size 8: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
  P(F) = 1/2
  E intersect F = { (THH), (TTT) }
  P(E intersect F) = 2 / 8 = 1/4
  Therefore
  P(E|F) = (1/4) | (1/2) = 1/2.

  - **Product Rule Form:** We often use this formula in the form:

  P(E intersect F) = P(F) * P(E|F)
- **Example: Sampling without replacement:** P (Ace on first draw and Ace on second draw) = (4/52)*(3/52)
- **Definition: Independent Events:** Two events E and F from a sample space S are independent if P(E intersect F) = P(E)*P(F).
- **Example:** In the coin tossing example are the specified two events independent?

  E = {THH, HTH, HHT, TTT}
  P(E) = 1/2
  F = {THH, THT, TTH, TTT}
  P(F) = 1/2
  E intersect F= { (THH), (TTT) }
  P(E intersect F) = 2/8 = 1/4
  P(E)*P(F) = 1/2 * 1/2 = 1/4
  So perhaps surprisingly yes!

Independent Bernouli trials

A sequence of independent Bernoulli trials satisfies the following conditions:
1. There are only two possible outcomes (success and failure) for each trial.
2. Each trial is repeated a certain number of times (Normally denote the number of trials by n)
3. Each trial is independent of every other trial.
4. The probability of success (p) and the probability of failure (q = 1 - p) are the same for each trial.

- EXAMPLE: Suppose we have a bag that contains 5 blue tickets and 15 red tickets. Draw out 8 tickets, with replacement, and record the number of times a red ticket appears.

Binomial Distribution

- Theorem 1gives us the formula for the probability of exactly r successes in n independent Bernoulli trials. (This is referred to as the Binomial Distribution).

  P(exactly r successes in n independent Bernoulli trials) = $C(n,r)p^r q^{(n-r)}$.

Exercise: Go to the Monty Hall page (linked below) -- play the game several times. Then read what the strategy should be.

The Monty Hall page