Sets

A set is an unordered collection of distinct objects.

A set can be represented by listing all of its elements in curly braces:

{a, b, c, d, e}

Set builder notation:

{ *x* | *P*(*x*) } is *the set of all x such that P(x).*

Set Equality

•Sets A and B are equal if and only if they contain the same elements.

•Examples:

A = {9, 2, 7, -3}, B = {7, 9, -3, 2} : A = B

A = {1, 2, 3},
B = {1, 2, 3, 4} : A ≠ B

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Set Properties

- Unordered:
  - No matter what objects a, b, and c denote, {a, b, c} = {a, c, b} = {b, a, c} = {b, c, a} = {c, a, b} = {c, b, a}.
- Distinct– duplicates are not allowed.

More Set Notation

- a∈A "a is a member of A"
- a∉A "a is not a member of A"
- ∅("null", "the empty set") is the set that contains no elements, i.e.∅= { }
- *A=B means that A and B have the same elements*
- *A*⊆*B* ("*A* is a subset of *B*") means that every element of *A* is also an element of *B*.
- *A*⊇*B* ("*A* is a superset of *B*") means *B*⊆*A*.
- *A*⊂*B* ("*A* is a proper subset of *B*") means that *A*⊆*B* but *A is not equal to B.*

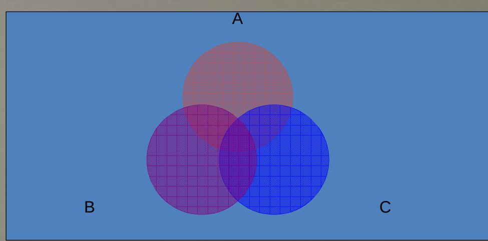
Examples of Sets and Set Notation

- {1, 2, 3, 4}
- {x | *x* is an integer where *x*>0 and *x*<5 }
- {*x* | *x* is a positive integer whose square is >0 and <25}

Cardinality of a Set

- |*A*| (read "the *cardinality* of *S*") is the number of elements in *A*.
- *Examples:*
- |∅|=0,
- |{1,2,3}| = 3,
- |{a,b}| = 2,
- |{{1,2,3},{4,5}}| = 2

Venn Diagrams



Link to Venn Diagram Practice

Showing Set Equality By Membership Tables

- 1 means "x is an element of this set"
- 0 means "x is not an element of this set"

A	B	C	$B \cap C$	$A \cup (B \cap C)$	$A \cup B$	$A \cup C$	$(A \cup B) \cap (A \cup C)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

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Set Operations

- *Union:* $A \cup B = \{x \mid x \in A \vee x \in B\}$.
- $\{2,3,5\} \cup \{3,5,7\} = \{2,3,5,7\}$
- *Intersection:* $A \cap B = \{x \mid x \in A \wedge x \in B\}$.
- $\{2, 4, 6, 8\} \cap \{3,4, 5, 6\} = \{4, 6\}$
- Two sets *A*, *B* are called **disjoint** if their intersection is empty. ($A \cap B = \emptyset$)
- Principle of Inclusion-Exclusion:
  $|A \cup B| = |A| + |B| - |A \cap B|$

Sets of Numbers

- Natural numbers **N** = {0, 1, 2, 3, …}
- Integers **Z** = {…, -2, -1, 0, 1, 2, …}
- Positive Integers $\mathbf{Z}^+$ = {1, 2, 3, 4, …}
- Real Numbers **R**
- Rational Numbers **Q (quotients of integers)**
- **Irrational Numbers: Real numbers that are not rational**

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Set Operations, Continued

- $A - B = \{x \mid x \in A \wedge x \notin B\}$
- $\{1,2,3,4,5,6\} - \{3,5,7,9\} = \{1,2,4,6\}$
- The *universe of discourse* (all objects under consideration) is noted by *U*.
- $\overline{A}$, *the complement* of *A*, is all elements in U which are not in A, i.e. $\overline{A}$ = U-A
  - Example:
  - U = **N**, A = {250, 251, 252, …}
  - $\overline{A}$ = {0, 1, 2, …, 248, 249}

The Power Set of a Set A

- The power set of A is the set of all subsets of A
- Examples:
- A = {x, y, z}
- Power set of A is {∅, {x}, {y}, {z}, {x, y}, {x, z}, {y, z}, {x, y, z}}
- Cardinality of the power set is $2^{|A|}$

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Cartesian Product of Two Sets

- For sets A, B, the *Cartesian product* $A \times B :\equiv \{(a, b) \mid a \in A \wedge b \in B\}$.
- *Example:* {a,b}×{1,2,} = {(a,1),(a,2),(b,1),(b,2)}
- For finite A, B, $|A \times B| = |A||B|$.
- Note that A ×B is not equal to B ×A

•The Cartesian product of two or more sets is defined as:

•$A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } 1 \leq i \leq n\}$