Sets

A set is an unordered collection of distinct objects.

A set can be represented by listing all of its elements in curly braces:

$$\{a, b, c, d, e\}$$

Set builder notation:

$\{ x \mid P(x) \}$ is *the set of all x such that P(x).*

Set Properties

- Unordered:
  - No matter what objects a, b, and c denote, {a, b, c} = {a, c, b} = {b, a, c} = {b, c, a} = {c, a, b} = {c, b, a}.
- Distinct– duplicates are not allowed.

Examples of Sets and Set Notation

- $\{1, 2, 3, 4\}$
- $\{x \mid x$ is an integer where $x>0$ and $x<5\}$
- $\{x \mid x$ is a positive integer whose square is >0 and $<25\}$

Set Equality

- Sets A and B are equal if and only if they contain the same elements.

- Examples:

A = {9, 2, 7, -3}, B = {7, 9, -3, 2} : A = B

A = {1, 2, 3},
B = {1, 2, 3, 4} : A $\neq$ B

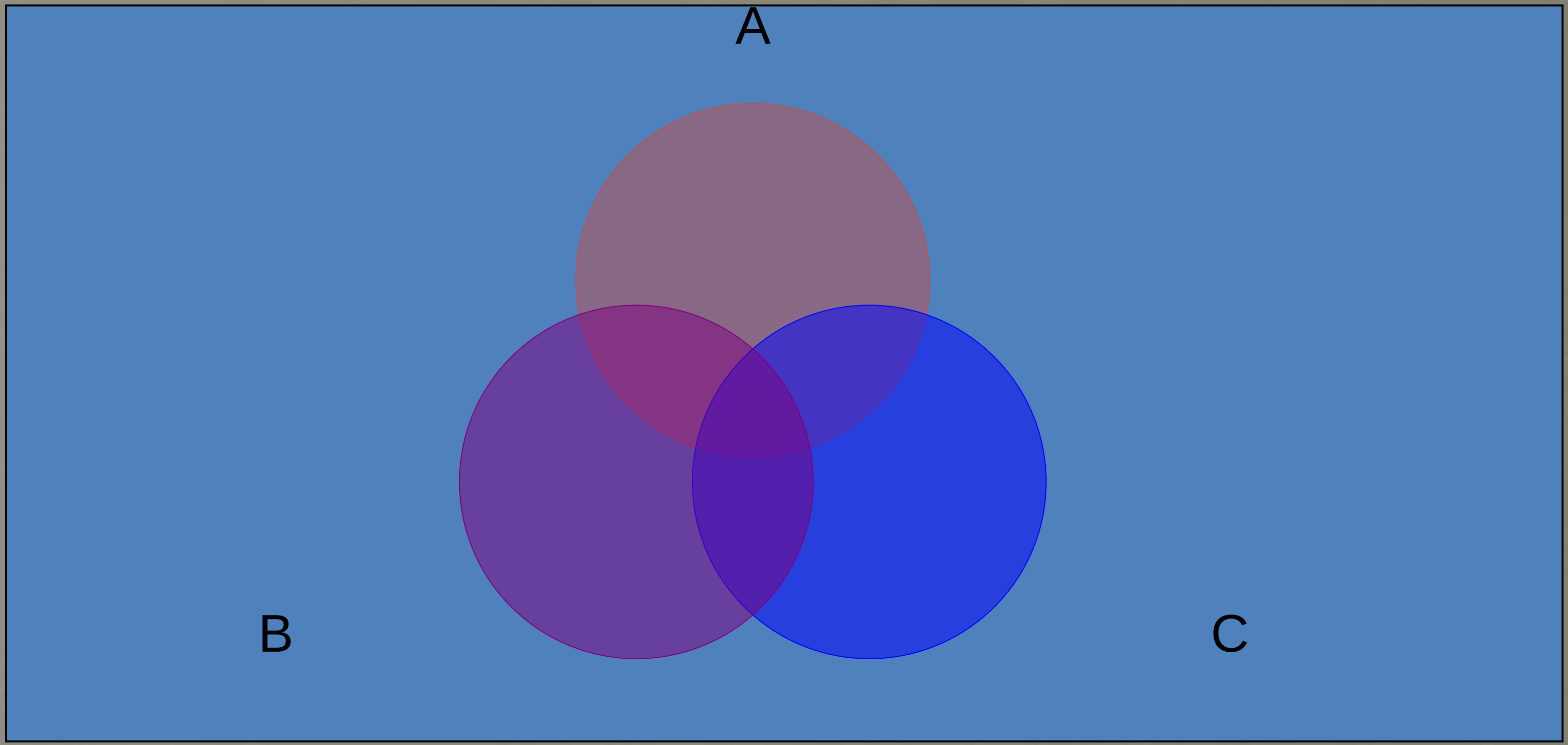
More Set Notation

- $a \in A$ "a is a member of A"
- $a \notin A$ "a is not a member of A"
- $\varnothing$("null", "the empty set") is the set that contains no elements, i.e. $\varnothing = \{\ \}$
- *A=B means that A and B have the same elements*
- $A \subseteq B$ ("*A* is a subset of *B*") means that every element of *A* is also an element of *B*.
- $A \supseteq B$ ("*A* is a superset of *B*") means $B \subseteq A$.
- $A \subset B$ ("*A* is a proper subset of *B*") means that $A \subseteq B$ but *A is not equal to B.*

Cardinality of a Set

- $|A|$ (read "the *cardinality* of *S*") is the number of elements in *A*.
- *Examples:*
- $|\varnothing| = 0$,
- $|\{1,2,3\}| = 3$,
- $|\{a,b\}| = 2$,
- $|\{\{1,2,3\},\{4,5\}\}| = 2$

Venn Diagrams



[Link to Venn Diagram Practice]

Set Operations

- *Union:* $A \cup B = \{x \mid x \in A \vee x \in B\}$.
- $\{2,3,5\} \cup \{3,5,7\} = \{2,3,5,7\}$
- *Intersection:* $A \cap B = \{x \mid x \in A \wedge x \in B\}$.
- $\{2, 4, 6, 8\} \cap \{3,4, 5, 6\} = \{4, 6\}$
- Two sets *A*, *B* are called **disjoint** if their intersection is empty. ($A \cap B = \emptyset$)
- Principle of Inclusion-Exclusion:

  $|A \cup B| = |\mathrm{A}| + |\mathrm{B}| - |A \cap B|$

Set Operations, Continued

- $A - B = \{x \mid x \in A \wedge x \notin B\}$
- $\{1,2,3,4,5,6\} - \{3,5,7,9\} = \{1,2,4,6\}$
- The *universe of discourse* (all objects under consideration) is noted by *U*.
- , t*he complement* of *A,* is all elements in U which are not in A, i.e. = U-A
  - Example:
  - U = **N**, A = {250, 251, 252, ...}
  - = {0, 1, 2, ..., 248, 249}

Showing Set Equality By Membership Tables

- 1 means "x is an element of this set"
- 0 means "x is not an element of this set"

A	B	C	$B \cap C$	$A \cup (B \cap C)$	$A \cup B$	$A \cup C$	$(A \cup B) \cap (A \cup C)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Sets of Numbers

- Natural numbers **N** = {0, 1, 2, 3, …}
- Integers **Z** = {…, -2, -1, 0, 1, 2, …}
- Positive Integers $\mathbf{Z}^+$ = {1, 2, 3, 4, …}
- Real Numbers **R**
- Rational Numbers **Q** **(quotients of integers)**
- **Irrational Numbers: Real numbers that are not rational**

The Power Set of a Set A

- The power set of A is the set of all subsets of A

- Examples:

- A = {x, y, z}
- Power set of A is {∅, {x}, {y}, {z}, {x, y}, {x, z}, {y, z}, {x, y, z}}

- Cardinality of the power set is $2^{|A|}$

Cartesian Product of Two Sets

- For sets *A*, *B*, the *Cartesian product* $A \times B :\equiv \{(a, b) \mid a \in A \wedge b \in B\}$.
- *Example:* $\{a,b\}\times\{1,2,\} = \{(a,1),(a,2),(b,1),(b,2)\}$
- For finite *A*, *B*, $|A\times B| = |A||B|$.
- Note that $A \times B$ is not equal to $B \times A$

•The Cartesian product of two or more sets is defined as:

•$A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } 1 \leq i \leq n\}$