Sequence

- A *sequence* $\{a_n\}$ is is a *function f* whose domain is a subset of N (the natural numbers {0,1,2,3,…}.
  - Often we have the domain is all of N or the all natural numbers greater than 0.
- We usually use the symbol a_n instead of $f(n)$, and call this the *nth term* of the sequence, and denote the sequence by $\{a_n\}$
- *Example:*

  $a_n = f(n) = 1/n$.

  $\{a_n\} = 1, 1/2, 1/3, \ldots$

Summation Notation

- Given a series $\{a_n\}$, an integer lower bound (or limit) j, and an integer upper bound k, then the summation of $\{a_n\}$ from j to k is written and defined as follows:

$$\sum_{i=j}^{k} a_i :\equiv a_j + a_{j+1} + \ldots + a_k$$

$$\begin{aligned}\sum_{i=2}^{4}(i^2+1) &= (2^2+1)+(3^2+1)+(4^2+1)\\ &= (4+1)+(9+1)+(16+1)\\ &= 5+10+17\\ &= 32\end{aligned}$$

Sum of the First *n* Integers

- The sum of the first n integers

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Sum of Geometric Progression

- The sum of the terms of a geometric progression (for r not equal to 1):

$$\sum_{j=0}^{n} ar^{j} = \frac{ar^{n+1} - a}{r - 1}$$

- Important special case: Let *a* = 1, *r* = 2

$$\sum_{j=0}^{n} 2^{j} = 2^{n+1} - 1$$

Geometric and Arithmetic Progressions

- A ***geometric progression*** is a sequence of the form a, ar, ar^2, ar^3, …, ar^n, … where $a, r \in \mathbf{R}$.
- This is like the real valued exponential function ar^x*, except that we restrict the domain to integers.*
- **Example:** a = initial investment of \$1000
- R = 1.05 (Assuming 5% annual interest rate)
- Models yearly growth of investment
- \$1000, \$1000·(1.05), \$1000 ·$(1.05)^2$, \$1000 ·$(1.05)^3$.

Arithmetic Progression

- An arithmetic progression is a sequence of the form

  $a, a + d, a + 2d, a + 3d, \ldots, a + nd, \ldots$

This is a discrete equivalent of linear growth.

- **Example:** Suppose you put $1000 under your mattress and add $100 each month.

- Here $a = 1000$, $d = 100$
- Money under the mattress at the end of each month:

  1000, 1000 + 100, 1000 + 2(100), etc.

Double Sums (Nested Sums)

$$\sum_{j=1}^{4}\sum_{i=1}^{3} ij = \sum_{j=1}^{4}\left(\sum_{i=1}^{3} ij\right) = \sum_{i=1}^{4}(1j + 2j + 3j)$$

$$= \sum_{j=1}^{4} 6j$$

$$= 6\cdot 1 + 6\cdot 2 + 6\cdot 3 + 6\cdot 4$$

$$= 60$$