

Functions

Section 2.3

Functions

For any sets A, B , we say that a *function* f from A to B (written $f: A \rightarrow B$) is a mapping that associates exactly one element $b = f(a)$ in B to each element a in A .

Example: Consider f the function that associates with each bit string the number of zero bits in the string.

$$f('1001100') = 3$$

$$f('111') = 0$$

A is called the domain of the function

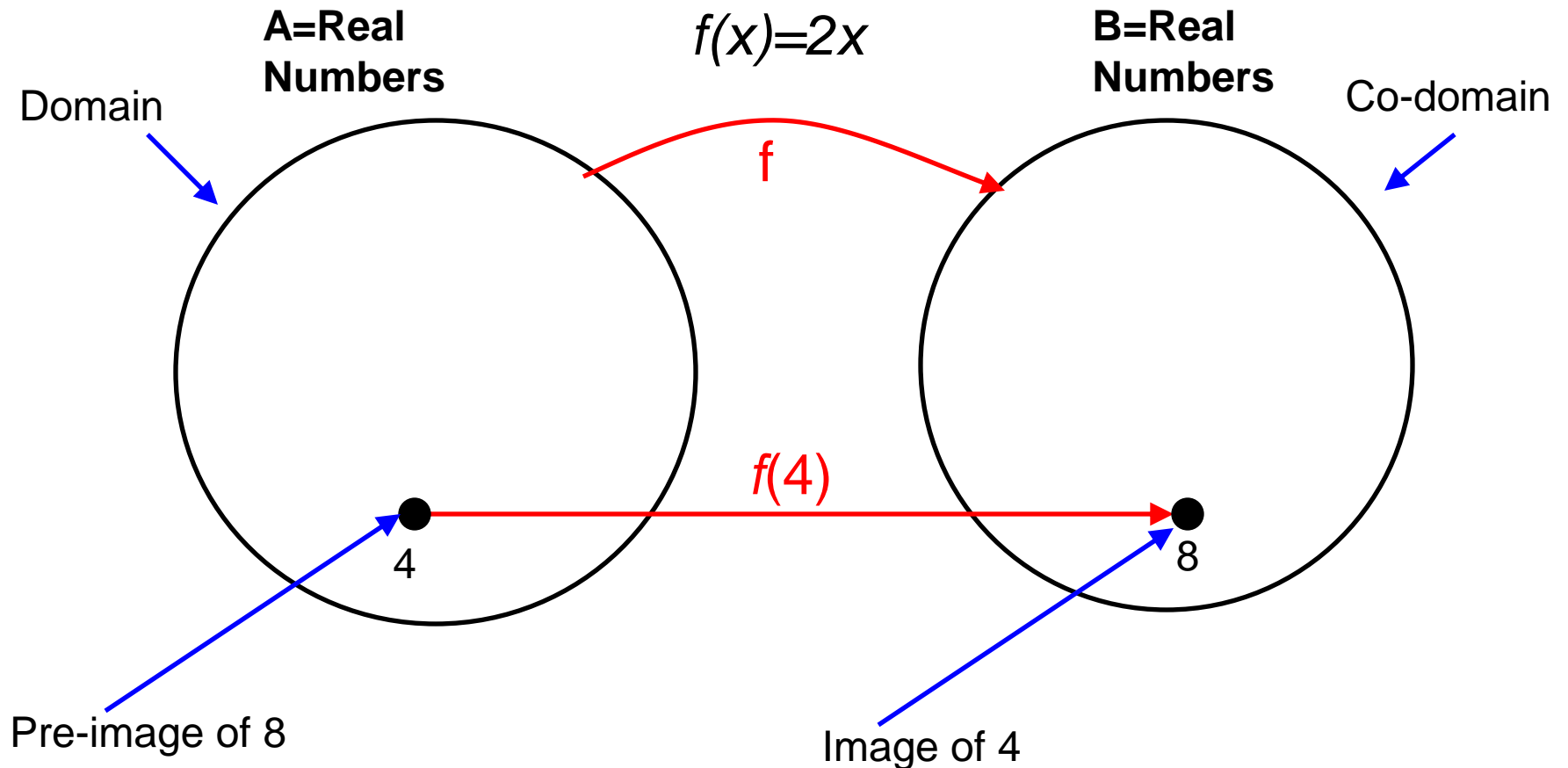
B is the *codomain* of f .

$b = f(a)$ is the *image* of a under f .

The *range* $R \subseteq B$ of f is $\{ b \mid \text{there exists an } a \text{ with } f(a)=b \}$.

i.e. The range of f is all possible values of the function.

Example of a Function



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$$f('1001100') = 3$$

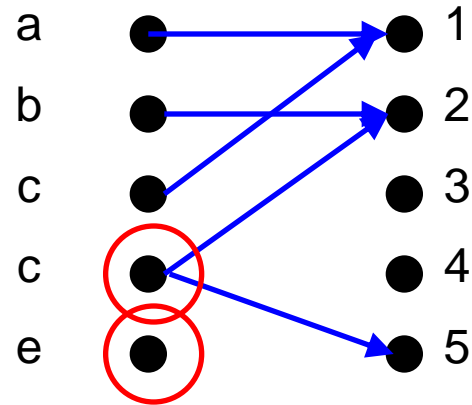
$$f('111') = 0$$

Domain: Set of all bit strings

Range: Set of nonnegative integers

Image of '10011001' is 3.

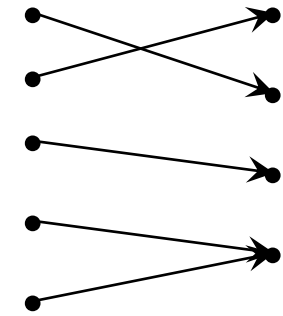
Not a Valid Function Example



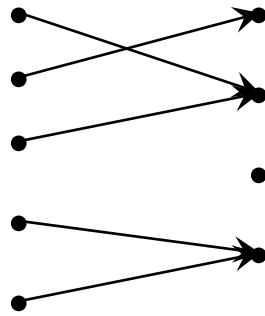
Injections and Surjections

- Def.: A function is *one-to-one* or *injective* or a *injection* if only one element of the domain is mapped to any given one element of the range.
- Def: A function $f:A \rightarrow B$ is *onto* or *surjective* or a *surjection* if its range is equal to its codomain.
- A function f is a *one-to-one correspondence*, or a *bijection*, if it is both one-to-one and onto.

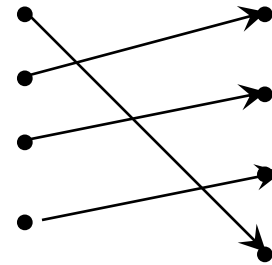
Categorizing Functions



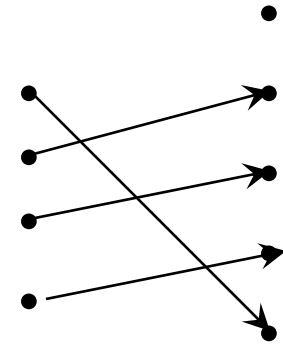
Onto
(but not 1-1)



Not Onto
(or 1-1)



Both 1-1
and onto



1-1 but
not onto

Inverse of a Function

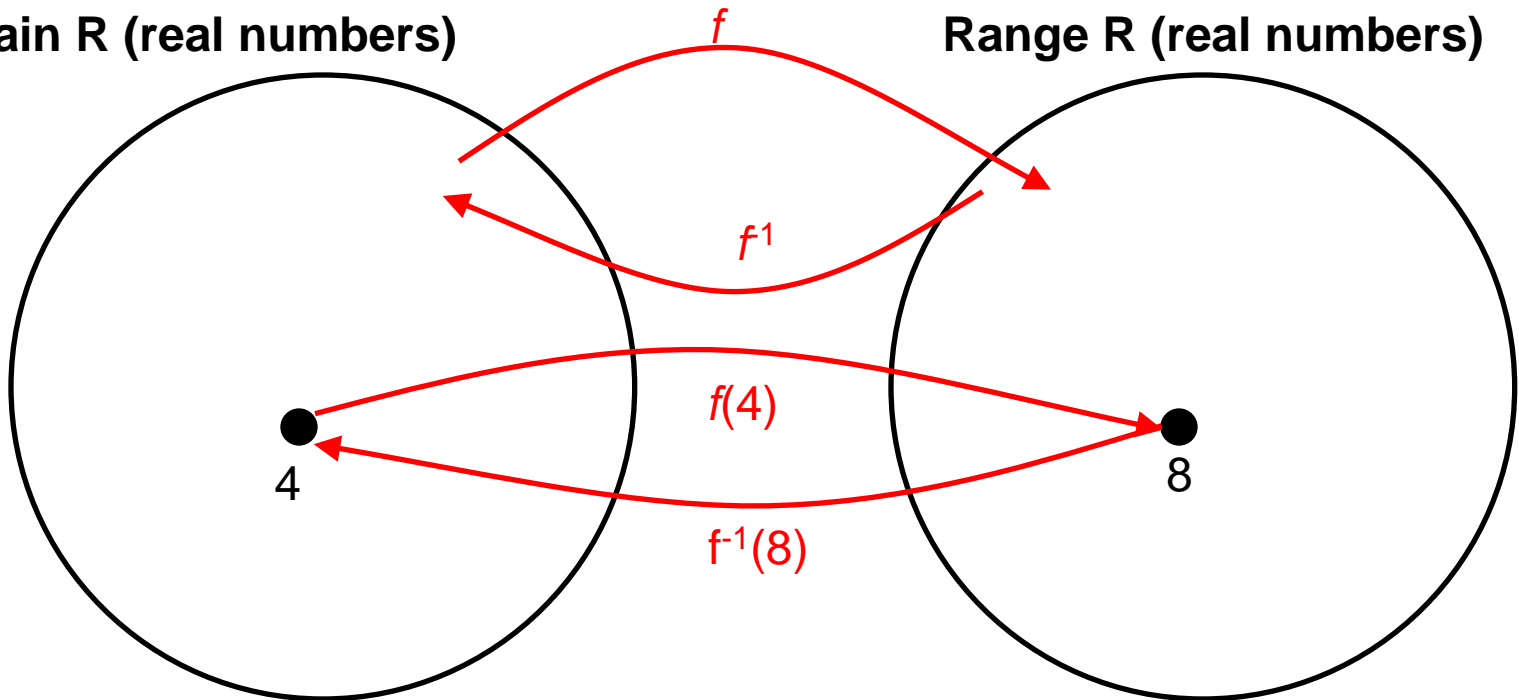
- A function f is a *one-to-one correspondence*, or a *bijection*, if it is both one-to-one and onto.
- For bijections $f:A \rightarrow B$, there exists an *inverse of f* , written $f^{-1}:B \rightarrow A$, which is the unique function such that
$$(f^{-1} \circ f)(x) = x$$

Inverse functions

$$f(x) = 2x$$

Domain \mathbb{R} (real numbers)

Range \mathbb{R} (real numbers)



$$f^{-1}(x) = x/2$$

Ceiling and Floor Functions

– $\lfloor x \rfloor$ (“floor of x ”) is the largest (most positive) integer $\leq x$.

$$\lfloor 2.3 \rfloor = 2 \quad \lfloor 2.8 \rfloor = 2 \quad \lfloor 2.0 \rfloor = 2 \quad \lfloor -2.3 \rfloor = -3$$

– $\lceil x \rceil$ (“ceiling of x ”) is the smallest (most negative) integer $\geq x$.

$$\lceil 2.3 \rceil = 3 \quad \lceil 2.8 \rceil = 3 \quad \lceil 3 \rceil = 3 \quad \lceil -2.3 \rceil = -2$$

Factorial Function

- Factorial is denoted by $n!$
- Defined by:
$$n! = 1 \cdot 2 \cdot 3 \dots (n-2) \cdot (n-1) \cdot (n)$$
- Example: $5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$
- $0!$ is defined to equal 1