Functions

Section 2.3

Example of a Function

Example: Consider f the function that associates with each bit string the number of zero bits in the string.

f('1001100') = 3
f('111') = 0

Domain: Set of all bit strings
Range: Set of nonnegative integers
Image of '10011001' is 3.

Functions

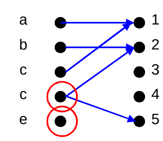
For any sets *A*, *B*, we say that a *function f from A to B* (written *f*: *A*→*B*) is a mapping that associates exactly one element *b* = *f*(*a*) in *B* to each element *a* in *A*.

Example: Consider f the function that associates with each bit string the number of zero bits in the string.

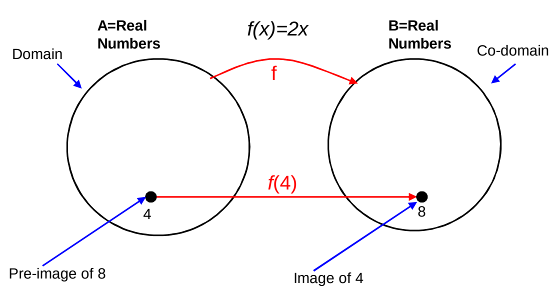
f('1001100') = 3
f('111') = 0

A is called the domain of the function
B is the *codomain* of *f*.
b = f(a) is the *image* of *a* under *f*.
The *range R*⊆*B* of *f* is { *b* | there exists an *a* with *f*(*a*)=*b* }.
i.e. The range of f is all possible values of the function.

Not a Valid Function Example



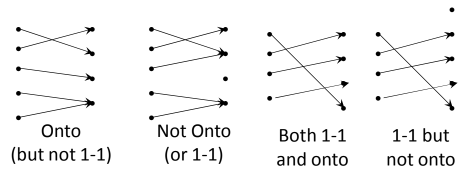
Example of a Function



Injections and Surjections

- Def.: A function is *one-to-one* or *injective* or a *injection* if only one element of the domain is mapped to any given one element of the range.
- Def: A function *f*:*A*→*B* is *onto* or *surjective* or *a surjection* if its range is equal to its codomain.
- A function *f* is *a one-to-one correspondence*, or *a bijection*, if it is both one-to-one and onto.

Categorizing Functions



Ceiling and Floor Functions

– $\lfloor x \rfloor$ ("floor of *x*") is the largest (most positive) integer $\leq x$.

$\lfloor 2.3 \rfloor = 2$ $\lfloor 2.8 \rfloor = 2$ $\lfloor 2.0 \rfloor = 2$ $\lfloor -2.3 \rfloor = -3$

– $\lceil x \rceil$ ("ceiling of *x*") is the smallest (most negative) integer $\geq x$.

$\lceil 2.3 \rceil = 3$ $\lceil 2.8 \rceil = 3$ $\lceil 3 \rceil = 3$ $\lceil -2.3 \rceil = 2$

Inverse of a Function

- A function *f* is *a one-to-one correspondence*, or *a bijection*, if it is both one-to-one and onto.
- For bijections $f:A \to B$, there exists an *inverse of f*, written $f^{-1}:B \to A$, which is the unique function such that $(f^{-1} \circ f)(x) = x$

Factorial Function

- Factorial is denoted by n!
- Defined by:
  n! = 1 ·2 ·3… (n-2) ·(n-1) ·(n)
- Example: 5! = 1 · 2 · 3 ·4 · 5= 120
- 0! is defined to equal 1

Inverse functions