The Limit of the Ratio of Successive Fibonacci Numbers

1. Restart and load the combinatorics library so we can access the fibonacci function:

```
> restart : with(combinat); with(plots)
```

[Chi, *bell*, binomial, *cartprod*, *character*, *choose*, *composition*, *conjpart*, *decodepart*, *encodepart*, *eulerian1*, *eulerian2*, *fibonacci*, *firstpart*, *graycode*, *inttovec*, *lastpart*, *multinomial*, *nextpart*, *numbcomb*, *numbcomp*, *numbpart*, *numbperm*, *partition*, *permute*, *powerset*, *prevpart*, *randcomb*, *randpart*, *randperm*, *setpartition*, stirling1, stirling2, *subsets*, *vectoint*]

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*]

2. Create the sequence of the first 100 fibonacci numbers:

```
> seq( [n, fibonacci(n) ], n = 1 ..100 )
```

[1, 1], [2, 1], [3, 2], [4, 3], [5, 5], [6, 8], [7, 13], [8, 21], [9, 34], [10, 55], [11, 89], [12, 144], [13, 233], [14, 377], [15, 610], [16, 987], [17, 1597], [18, 2584], [19, 4181], [20, 6765], [21, 10946], [22, 17711], [23, 28657], [24, 46368], [25, 75025], [26, 121393], [27, 196418], [28, 317811], [29, 514229], [30, 832040], [31, 1346269], [32, 2178309], [33, 3524578], [34, 5702887], [35, 9227465], [36, 14930352], [37, 24157817], [38, 39088169], [39, 63245986], [40, 102334155], [41, 165580141], [42, 267914296], [43, 433494437], [44, 701408733], [45, 1134903170], [46, 1836311903], [47, 2971215073], [48, 4807526976], [49, 7778742049], [50, 12586269025], [51, 20365011074], [52, 32951280099], [53, 53316291173], [54, 86267571272], [55, 139583862445], [56, 225851433717], [57, 365435296162], [58, 591286729879], [59, 956722026041], [60, 1548008755920], [61, 2504730781961], [62, 4052739537881], [63, 6557470319842], [64, 10610209857723], [65, 17167680177565], [66, 27777890035288], [67, 44945570212853], [68, 72723460248141], [69, 117669030460994], [70, 190392490709135], [71, 308061521170129], [72, 498454011879264], [73, 806515533049393], [74, 1304969544928657], [75, 2111485077978050], [76, 3416454622906707], [77, 5527939700884757], [78, 8944394323791464], [79, 14472334024676221], [80, 23416728348467685], [81, 37889062373143906], [82, 61305790721611591], [83, 99194853094755497], [84, 160500643816367088], [85, 259695496911122585], [86, 420196140727489673], [87, 679891637638612258], [88, 1100087778366101931], [89, 1779979416004714189], [90, 2880067194370816120], [91, 4660046610375530309], [92, 7540113804746346429], [93, 12200160415121876738], [94, 19740274219868223167], [95, 31940434634990099905], [96, 51680708854858323072], [97, 83621143489848422977], [98,

$$13530185234470674604 9], [99, 218922995834555169026], [100, 354224848179261915075]$$

3. Look at the ratio of the fourth Fibonacci number to the third:

> $\frac{fibonacci(4)}{fibonacci(3)}$

$$\frac{3}{2}$$

4. Create the sequence of the ratios of successive fibonacci numbers up to n = 50:

> $fibratio := \left[seq\left(\left[n, \frac{fibonacci(n)}{fibonacci(n-1)} \right], n = 2..50 \right) \right]$

$$fibratio := \left[[2, 1], [3, 2], \left[4, \frac{3}{2}\right], \left[5, \frac{5}{3}\right], \left[6, \frac{8}{5}\right], \left[7, \frac{13}{8}\right], \left[8, \frac{21}{13}\right], \left[9, \frac{34}{21}\right], \left[10, \frac{55}{34}\right], \left[11, \frac{89}{55}\right], \left[12, \frac{144}{89}\right], \left[13, \frac{233}{144}\right], \left[14, \frac{377}{233}\right], \left[15, \frac{610}{377}\right], \left[16, \frac{987}{610}\right], \left[17, \frac{1597}{987}\right], \left[18, \frac{2584}{1597}\right], \left[19, \frac{4181}{2584}\right], \left[20, \frac{6765}{4181}\right], \left[21, \frac{10946}{6765}\right], \left[22, \frac{17711}{10946}\right], \left[23, \frac{28657}{17711}\right], \left[24, \frac{46368}{28657}\right], \left[25, \frac{75025}{46368}\right], \left[26, \frac{121393}{75025}\right], \left[27, \frac{196418}{121393}\right], \left[28, \frac{317811}{196418}\right], \left[29, \frac{514229}{317811}\right], \left[30, \frac{832040}{514229}\right], \left[31, \frac{1346269}{832040}\right], \left[32, \frac{2178309}{1346269}\right], \left[33, \frac{3524578}{2178309}\right], \left[34, \frac{5702887}{3524578}\right], \left[35, \frac{9227465}{5702887}\right], \left[36, \frac{14930352}{9227465}\right], \left[37, \frac{24157817}{14930352}\right], \left[38, \frac{39088169}{24157817}\right], \left[39, \frac{63245986}{39088169}\right], \left[40, \frac{102334155}{63245986}\right], \left[41, \frac{165580141}{102334155}\right], \left[42, \frac{267914296}{165580141}\right], \left[43, \frac{433494437}{267914296}\right], \left[44, \frac{701408733}{433494437}\right], \left[45, \frac{1134903170}{701408733}\right], \left[46, \frac{1836311903}{1134903170}\right], \left[47, \frac{2971215073}{1836311903}\right], \left[48, \frac{4807526976}{2971215073}\right], \left[49, \frac{7778742049}{4807526976}\right], \left[50, \frac{12586269025}{7778742049}\right] \right]$$

5. Approximate this ratios to 20 decimal places:

> $evalf(fibratio, 20)$

$$[[2., 1.], [3., 2.], [4., 1.5000000000000000000], [5., 1.6666666666666666667], [6., 1.6000000000000000000], [7., 1.6250000000000000000], [8., 1.6153846153846153846], [9., 1.6190476190476190476], [10., 1.6176470588235294118], [11., 1.6181818181818181818], [12., 1.6179775280898876404], [13., 1.6180555555555555556], [14., 1.6180257510729613734], [15., 1.6180371352785145889], [16., 1.6180327868852459016], [17., 1.6180344478216818642], [18., 1.6180338134001252348], [19., 1.6180340557275541796], [20., 1.6180339631667065295], [21., 1.6180339985218033999], [22., 1.6180339850173579390], [23., 1.6180339901755970866], [24., 1.6180339882053250515], [25., 1.6180339889579020014], [26., 1.6180339886704431856], [27., 1.6180339887802426829], [28., 1.6180339887383030069], [29., 1.6180339887543225376], [30., 1.6180339887482036213], [31., 1.6180339887505408394], [32., 1.6180339887496481015], [33.,$$

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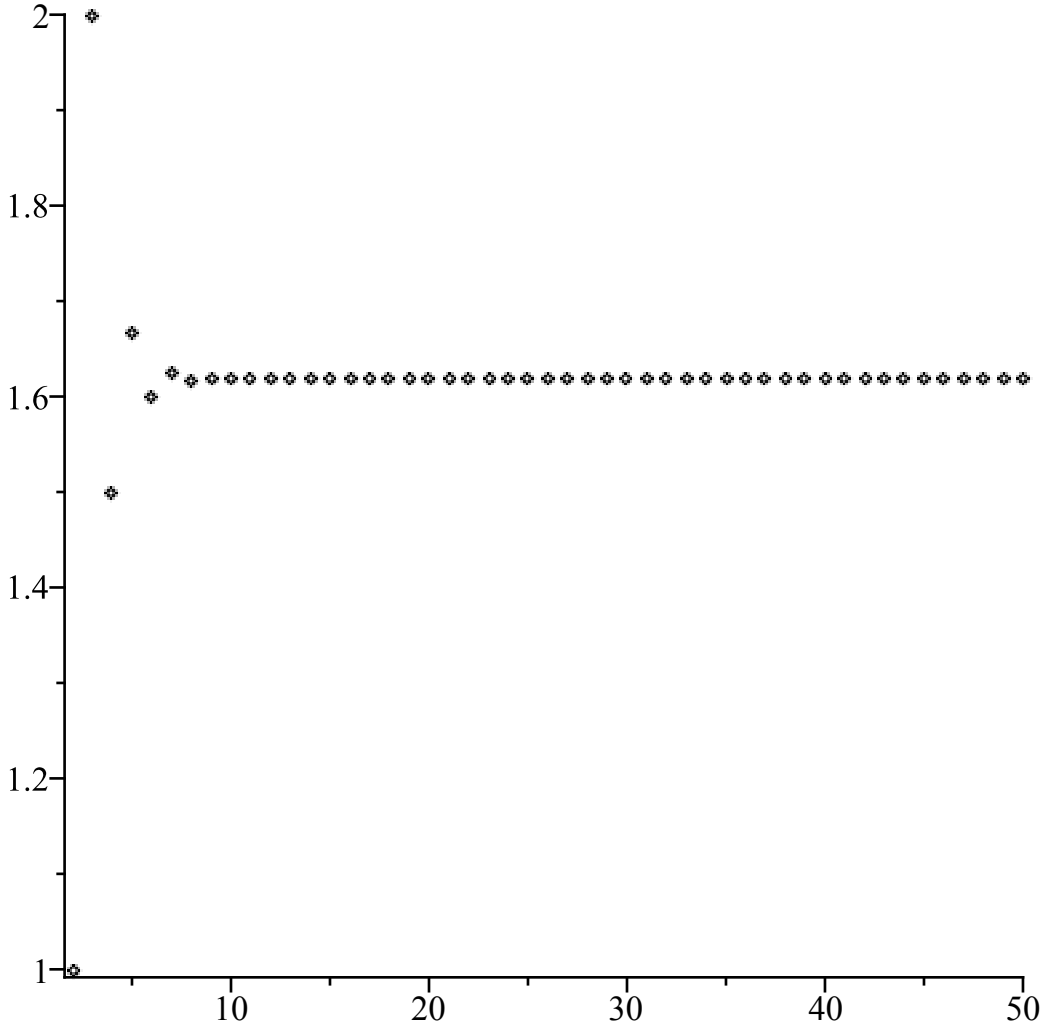
1.6180339887499890970], [34., 1.6180339887498588484], [35.,
1.6180339887499085989], [36., 1.6180339887498895959], [37.,
1.6180339887498968544], [38., 1.6180339887498940819], [39.,
1.6180339887498951409], [40., 1.6180339887498947364], [41.,
1.6180339887498948909], [42., 1.6180339887498948319], [43.,
1.6180339887498948544], [44., 1.6180339887498948458], [45.,
1.6180339887498948491], [46., 1.6180339887498948479], [47.,
1.6180339887498948483], [48., 1.6180339887498948482], [49.,
1.6180339887498948482], [50., 1.6180339887498948482]]
```

6. Plot the sequence -- load plot library first so we can use pointplot:

```
> with(plots)
[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d,
    conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot,
    display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot,
    implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot,
    listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple,
    odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d,
    polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions,
    setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]
> pointplot( fibratio )
```



7. Evaluate $\frac{1+\sqrt{5}}{2}$ to 20 decimal places.

```
> evalf((1+sqrt(5))/2, 20)
```

1.6180339887498948482

8. Verifying some of the golden ratio properties (See exercise 19, 20 page 622)

```
> p := evalf((1+sqrt(5))/2, 20)
```

$p := 1.6180339887498948482$

```
> 1/p; p-1
```

0.6180339887
0.618033989

```
> p^2
```

2.618033990

Lucas Numbers: These are like the Fibonacci numbers, except the first two numbers are 1 and 3:

> $lucas := n \rightarrow lucas(n-1) + lucas(n-2)$

$$lucas := n \rightarrow lucas(n-1) + lucas(n-2)$$

> $lucas(1) := 1; lucas(2) := 3$

$$lucas(1) := 1$$
$$lucas(2) := 3$$

> $seq([n, lucas(n)], n = 1..10)$

$$[1, 1], [2, 3], [3, 4], [4, 7], [5, 11], [6, 18], [7, 29], [8, 47], [9, 76], [10, 123]$$

> $lucasratio := \left[seq\left(\left[n, \frac{lucas(n)}{lucas(n-1)}\right], n = 2..20\right)\right]$

$$lucasratio := \left[[2, 3], \left[3, \frac{4}{3}\right], \left[4, \frac{7}{4}\right], \left[5, \frac{11}{7}\right], \left[6, \frac{18}{11}\right], \left[7, \frac{29}{18}\right], \left[8, \frac{47}{29}\right], \left[9, \frac{76}{47}\right], \left[10, \frac{123}{76}\right], \left[11, \frac{199}{123}\right], \left[12, \frac{322}{199}\right], \left[13, \frac{521}{322}\right], \left[14, \frac{843}{521}\right], \left[15, \frac{1364}{843}\right], \left[16, \frac{2207}{1364}\right], \left[17, \frac{3571}{2207}\right], \left[18, \frac{5778}{3571}\right], \left[19, \frac{9349}{5778}\right], \left[20, \frac{15127}{9349}\right]\right]$$

> $evalf(lucasratio)$

$$[[2., 3.], [3., 1.333333333], [4., 1.750000000], [5., 1.571428571], [6., 1.636363636], [7., 1.611111111], [8., 1.620689655], [9., 1.617021277], [10., 1.618421053], [11., 1.617886179], [12., 1.618090452], [13., 1.618012422], [14., 1.618042226], [15., 1.618030842], [16., 1.618035191], [17., 1.618033530], [18., 1.618034164], [19., 1.618033922], [20., 1.618034014]]$$

>