MAT – 111
Equation sheet for Angular Velocity and Newton's Universal Law of Gravitation

θ = s / r
s = arc length
r = radius

1 radian = approximately 57.3 degrees
Ω = angular velocity in radians
α = angular acceleration in radians per second squared

$\Omega = \theta$ / t
Ω = 2 * Pi * f
f = number of revolutions per second
θ = 2 Pi * number of revolutions

$\alpha = (\Omega_f - \Omega_0) / t$
$\Omega = (\Omega_f + \Omega_0) / 2$
$\theta = \Omega * t = t * (\Omega_f + \Omega_0) / 2$
$\theta = \Omega_0 * t + 1 / 2 * \alpha * t^2$
$\theta = \Omega_f * t - 1 / 2 * \alpha * t^2$
$2 * \alpha * \theta = \Omega_f^2 - \Omega_0^2$

v = (θ * r) / t
v = Ω * r

Symbol for tangential acceleration is at.
at = $(v_f - v_0) / t$
at = α * r

Symbol for centripetal acceleration is ac.
ac = v^2 / r

equations for kinetic energy and moment of inertia
I = 1 / 2 * m * r^2
ek = 1 / 2 * mv^2 = 1 / 2 * Ω^2 * r^2
I = moment of inertia
I = $mr_1^2 + mr_2^2 + mr_3^2 + \ldots$
Ek = 1 / 2 * I * Ω^2

Equations for torque and radius of gyration
For equation I = mk^2, k = radius of gyration.

$I = mk^2$
$At = \alpha * r$
$f = m * At = m * \alpha * r$
$f * r = mr^2 * \alpha$
$T = mr^2 * \alpha$
$T = I * \alpha$
$T = fr$

equations for work and power
$\Omega = f * s$
$\Omega = f * r\theta$
$\Omega = T * \theta$
$p = \Omega / t = (T\theta) / t$

equations for angular momentum
$L = mvr$
$mvr = m * \Omega^2 = mr^2 * \Omega$
$L = I * \Omega$
$T * t = I * \Omega_f - I * \Omega_0$

equations for Newton's law of gravitation
$f = G * (m_1 * m_2) / r^2$
$G = 6.67 * 10^{-11}$ n * m^2 / kg^2 = $3.44 * 10^{-8}$ lb * ft^2 / slug^2
$w = m * g = G * (m_1 * m_2) / r^2$

Example exercises

1.
Give the measurement in radians of an angle produced by an arc length of 40 cm and a radius of 15 cm.
Answer: 2.667 radians

2.
A bicycle wheel with a radius of 78 cm makes 75 revolutions in 2 minutes. What are its angular velocity and its linear displacement?
Answers
Part 1: angular velocity is 3.93 radians per second
Part 2: 471.24 radians

3.
The speed of gravity on Mars is 3.71 m / s^2 and its equatorial radius is $3.4 * 10^6$ m. What is its mass?
Answer: $6.43 * 10^{23}$ kg

4.
A hypothetical planet has gravitational speed of 5.7 m / s^2 and its equatorial radius is 7.6 * 10^6 m. What is its mass?
Answer: 4.94 * 10^24 kg

5.
Find the rotational kinetic energy for the following objects with the following radii from the center of rotation. This system is rotating with an angular velocity of 23 radians per second.

Mass of object in kg	Radius from center of rotation in meters
3	.5
4	1.2
3.4	14.5
2.2	1.6
3.1	10.3
4.5	7.4
5.2	9.2

Answer: 460,869.83 J

6.
Find the radius of gyration for a rotating body with a mass of 3 kg and a moment of inertia of 6 kg m^2.
Answer: 1.41 meters

7.
A wheel with a radius of 4 meters and a mass of 35 kg is rotating at a rate of 915 revolutions per minute. What force applied tangent to the edge of the wheel will stop its rotation in 45 seconds? What is the rate of acceleration once the force is applied?
Answers
Part 1: angular acceleration is -2.13 radians per second squared
Part 2: resulting force is -149.05 n