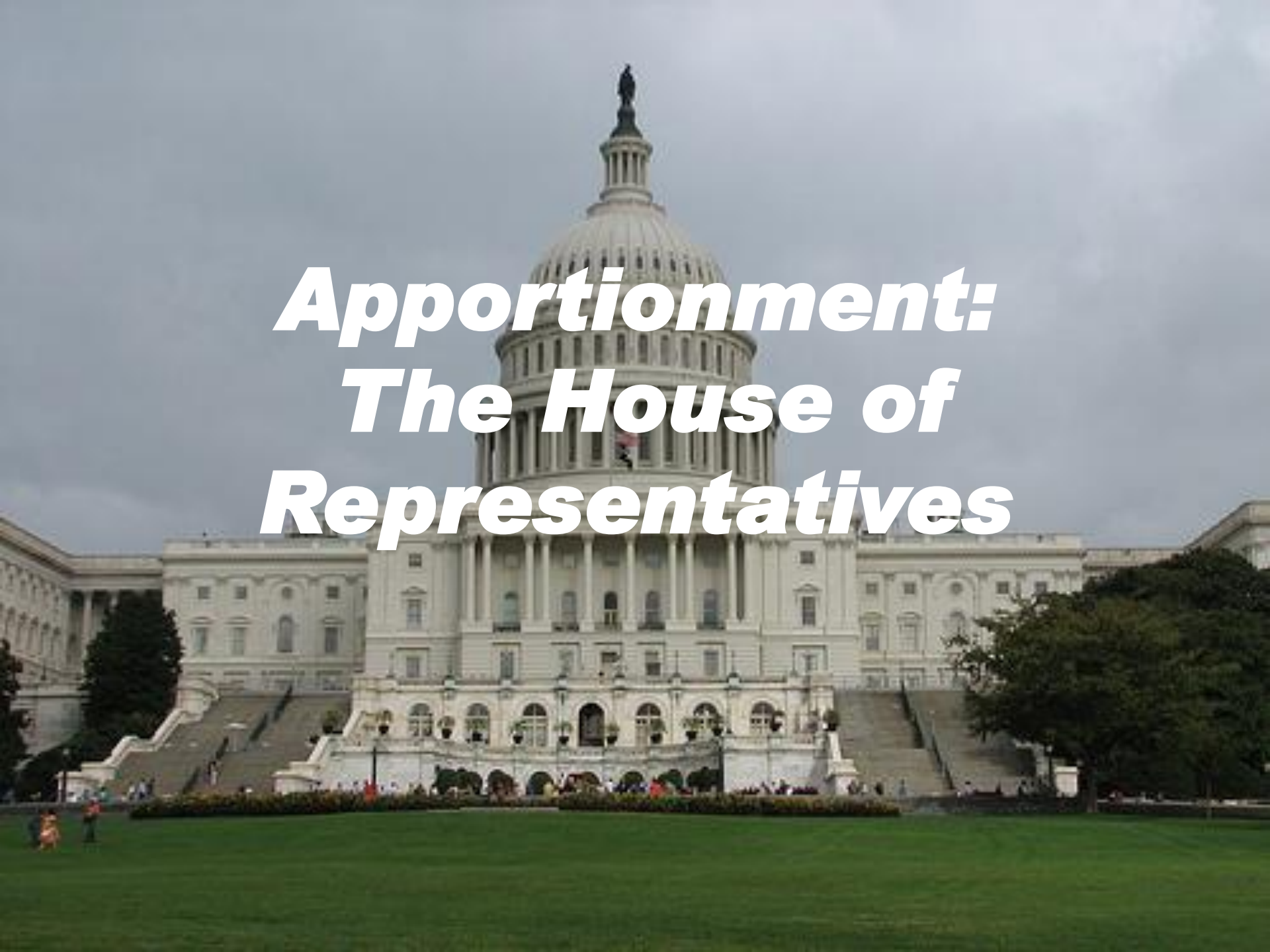
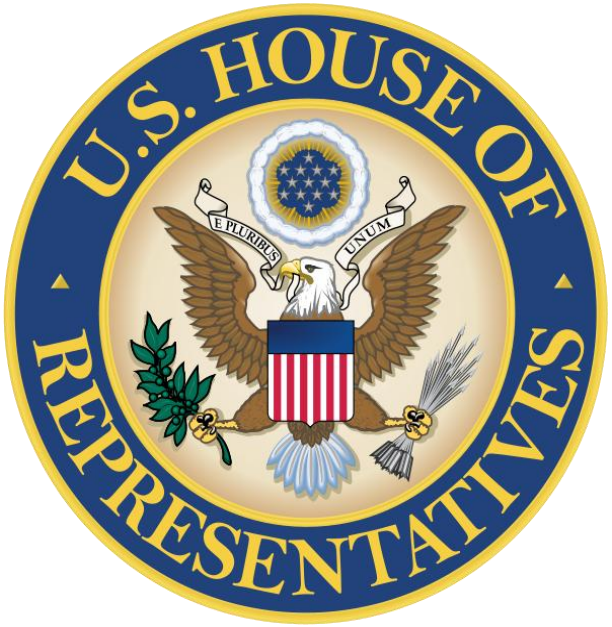


Apportionment: The House of Representatives

What is Apportionment?

- Apportionment is a process used to divide the available seats among the states.

- Generally, apportionment is a process used to divide any set of people or objects among various individuals or groups.



Background

- The mathematics of apportionment arose from historical attempts to meet the needs of the United States Constitution.
- Legislative Branch
  - House of Representatives
    - 435 seats, apportioned to the states according to their populations.
    - Allows Congress to set the total # of representatives as long as the total number does not exceed one for each 30,000 people.
  - Senate
    - Each state gets two senators, 100 total.

FACT

- 2000 Apportionment
  - Changed the House representation of 18 states.
  - Arizona, Georgia, Florida, and Texas gained 2 seats each.
  - Nevada, Colorado, North Carolina, and California each gained 1 seat.
  - New York, Pennsylvania each lost 2 seats.
  - Mississippi, Oklahoma, Indiana, Wisconsin, Illinois, Michigan, Ohio, and Connecticut each lost one seat.

Problem with Apportionment

- When using the math used in apportionment some states become over represented while some become under represented
  - Example: Rhode Island has 1,050,000 people. Which equals 1.6 x 646,000.
    - Does Rhode Island get one representative or two representatives?

Standard Divisor

- **Standard Divisor** is the average number of people per seat for the entire population of the United States.
  - only one standard divisor
- Standard Divisor= total U.S. population ÷ number of seats
- Standard Divisor for the U.S. is 646,000 people

Standard Quota

- **Standard Quota** for a state is the number of seats it would be entitled to if fractional seats were allowed.
- Standard Quota = state population ÷ standard divisor
- Determines the number of representatives each state have

Not just with Apportionment

- Standard Divisor and Quota can be applied with anything.
  - Replace the number of seats with the things being apportioned and state populations with the appropriate populations for the problem.

Standard Quota

- Standard Quota for Montana

  Standard Quota=state population/standard divisor

  902,000/646,000=1.4

- Montana has only one representative, which means in Congress they are under represented. Proving the problem with apportionment

Hamilton's Method

- First find the standard quota for each state
- Then give each state the number of seats by rounding the standard quota down. (minimum quota)
- The extra seats go to the states with the highest fractional remainders. Give the states the seats until they are all gone.

Applying the Method

State	A	B	C	D	Total
Population	936	2726	2603	3735	10,000
Standard Quota	9.36	27.26	26.03	37.35	100
Minimum Quota	9	27	26	37	99
Fractional Remainder	.36 (largest)	.26	.03	.35	1
Final Apportionment	10	27	26	37	100

State A would get the last seat because it has the highest fractional remainder.

Problems with Hamilton's Method

- Three Paradoxes:
  - Alabama Paradox
  - Population Paradox
  - New States Paradox

Alabama Paradox

- Most famous problem that arose.
- After 1880 census, Chief Clerk of the Census Office used Hamilton's Method for various possible House sizes.
- RESULT: Alabama would get 8 seats in a House of 299 representatives, and only 7 seats in a House of 300 representatives.
- Unfair because Alabama loses a seat when the total number of seats increases.
- Outcome: Alabama didn't lose a seat in 1882 Apportionment because Congress chose a larger house size, in which the paradox didn't present itself, then abandoned in 1900 by Congress.

Population Paradox

- Found when Hamilton's Method would give a seat to Maine at Virginia's expense even though Virginia was growing much faster than Maine.

New States Paradox

- Discovered in 1907-Oklahoma became 46th state in United States, not yet time for reapportionment, Congress added NEW seats for Oklahoma.
- Based on Population, Oklahoma was entitled to 5 seats. Congress increased House seats from 386 to 391.
- Outcome: Calculations with Hamilton's Method showed that adding the new 5 seats would cause New York to lose a seat while Maine gaining one.

Summary of Paradoxes

- ***Alabama Paradox*** occurs when the total number of available seats increases, yet one state (or more) loses seats as a result.
- When a slow-growing state gains a seat at the expense of a faster-growing state- we have the ***population paradox***.
- When additional seats are added to accommodate a new state, we do not expect this addition to change the apportionment for existing states the ***new states paradox*** occurs.

First Presidential Veto

- 1791-Both the Senate and House voted to adopt Hamilton's Method for apportionment.
- 1792- Bill authorizing Hamilton's Method received the first presidential veto in the history of the United States.
- Congress unable to override the veto-instead passed a bill authorizing a different apportionment method proposed by Thomas Jefferson.

Jefferson's Method

- Changed the Standard quotas to new values called modified quotas, so that the new set of minimum quotas would use up all the seats.

If there are extra seats, star the computations over:

1. Choose modified that is lower than the standard divisor. Use it to find modified quotas.

Modified Quota=state population/modified divisor

2. Round Modified Quotas down to find new set of minimum quotas.

Jefferson's Method continued

- If there are just enough seats to fill all the minimum quotas, the apportionment is done.
- If not, do one of the following:
  - If there are still extra seats with the new minimum quotas, start again with a lower modified divisor.
  - If there are not enough total seats to fill all the minimum quotas, start again with a higher modified divisor (but still smaller than the standard divisor).

Applying Jefferson's Method to the Four-State Date Table

STATE	A	B	C	D	Total
Population	936	2726	2603	3735	10,000
Standard Quota	9.36	27.26	26.03	37.35	100
Minimum Quota	9	27	26	37	99
Modified Quota with divisor 99)	9.45	27.54	26.29	37.73	101.01
Minimum Quota (with divisor 99)	9	27	26	37	99
Modified Quota (with divisor 98)	9.55	27.82	26.56	38.11	102.04
Minimum Quota (with divisor 98)	9	27	26	38	100

Hill-Huntington Method

- Method used currently
- Named after Joseph Hill who served as Chief Statistician of the Census Bureau and Harvard mathematician Edward Huntington.
- Important Key: Rounding is based on the geometric mean of the integers on either side of the modified quota.

Geometric Mean of any two numbers is ***x*** and ***y*** is the square root of (x * y).

-Modified Quota is less than the geometric mean of two nearest integers it gets rounded down.

-Example:

If a state had a modified quota of 2.47, in Webster's Method you would round to either 2 or 3. In this method though you find the geometric mean of 2 and 3 which is…

$\sqrt{}(2*3)= \sqrt{}6 \approx 2.45$

-Since 2.47 is greater than the geometric mean, it gets rounded up to 3.

Method increases the chance that extra seats will go to smaller states rather than larger states.