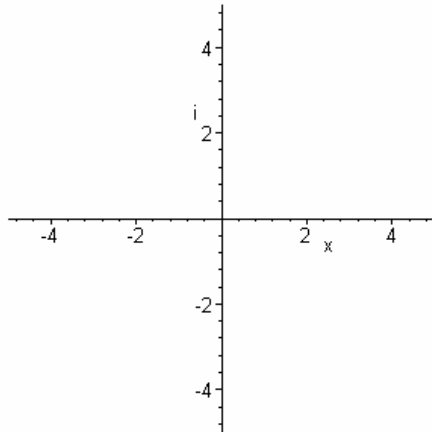


# MTH 111 Mathematics as a Human Pursuit Worksheet on the Fractal Geometry of the Mandelbrot Set

Name \_\_\_\_\_



0. First graph these complex numbers  $a$ ,  $b$ ,  $c$ , and  $d$  on the plot above. Then perform the following arithmetic on these complex numbers:

$$a = 3 - 2i \quad b = -1 + 3i \quad c = 4i \quad d = 3$$

a)  $a + b$

d)  $b^2$

b)  $a - b$

e)  $a * b$

c)  $a^2$

f)  $a * c$

1. Give the precise definition of the Mandelbrot set.

2. Give the precise definition of the Julia set.

3. Explain what the following concepts are:

a) an iterative function

b) a seed for an iterative function

c) the orbit of an iterative function and seed.

4. Compute the orbit of the following seeds with the given iterative functions. Note whether the orbit goes to a fixed point, cycles (if so give its period), escapes to infinity, or is chaotic (does not tend to infinity, but does not have a predictable pattern.)

a) seed:  $x_0 = 0$ ; iterative function  $f(x) = x^2$ .

b) seed:  $x_0 = 0$ ; iterative function  $f(x) = x^2 - 3$ .

c) seed:  $x_0 = 0$ ; iterative function  $f(x) = 1/(x+2)$ .

d) seed:  $x_0 = i$ ; iterative function  $x = x^2 - i$ .

5. Compute the orbit of 0 under the  $x^2 + c$  for the given value of  $c$ . State what the fate of the orbit is (as in problem 4 -- fixed point, cycles with period \_\_\_\_, escapes to infinity, or chaotic). Use a calculator or Maple to compute sufficient iterations to illustrate. Then tell whether the  $c$  is a member of the Mandelbrot set or not, based on your result. Finally plot and label  $c$  on the complex plane as a solid black dot if in the Mandelbrot set, and as a solid red dot if not. (Use one graph -- plot each on the same plot.)

a)  $c = 2i$

b)  $c = -i$

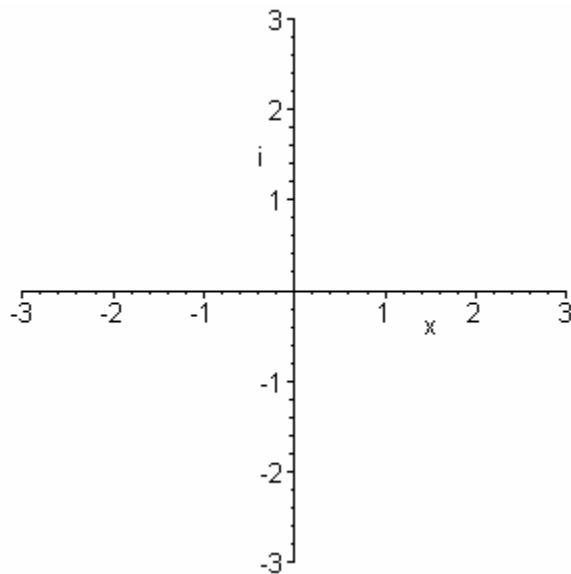
c)  $c = .12 + 0.75i$

d)  $c = -.12 + 0.75i$

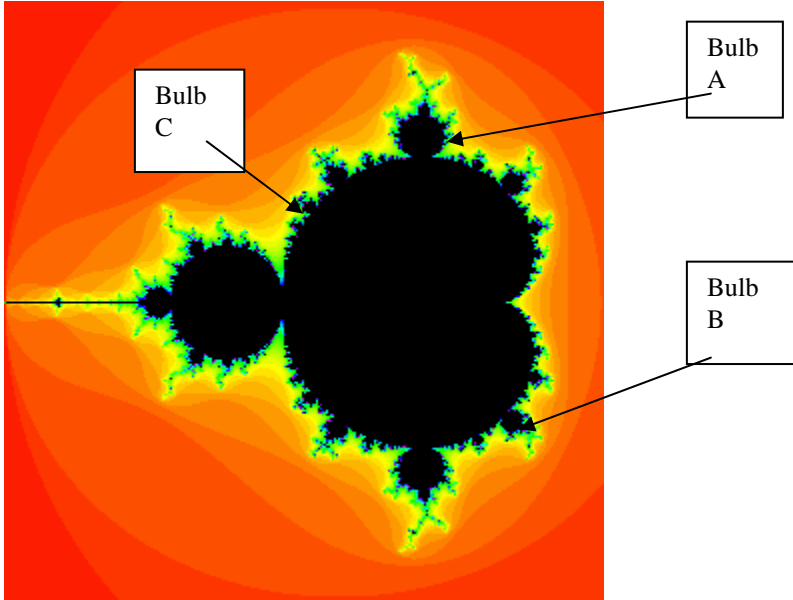
e)  $c = 0.28 + 0.54i$

f)  $c = 0.39 + 0.22i$

Plots:



6. For the Mandelbrot set below, find a) the period of the orbit and b) the fraction  $p/q$  associated with the indicated bulbs. Justify your answer in terms of antennae or Julia sets. Use the The Fractal Microscope (Interactive exploration of the Mandelbrot Set -- linked on our class web page) to help you find the answers.



Bulb A:

Bulb B:

Bulb C: