

Loan Amortization

A loan, such as a mortgage, is **amortized** when part of each installment payment is used to pay interest and the remaining part is used to reduce the principal.

These are one of the most important forms of an annuity..

An **amortization schedule** is a table that provides an analysis of each payment period -- including the following:

1. Principal outstanding at the beginning of the time period
2. Interest for that time period
3. Payment at the end of that period
4. Principal repaid at the end of that period.

(There also may be running sums of 2-4 computed and reported in the table.)

Formulas

The following formulas are used to calculate these entries:

Periodic Payment

Suppose A denotes the starting loan amount. A is the present value of the annuity.

Then the periodic payment R is given by the formula

$$R = A \frac{r}{1 - (1 - r)^{-n}}$$

where r is the periodic interest rate and n is the number of periods.

Principal Outstanding

The principal outstanding at the beginning of the kth time period is given by the formula

$$PO = R \frac{1 - (1 + r)^{-n+k-1}}{r}$$

Interest on kth Payment

The interest in the kth payment is given by the formula

$$\begin{aligned} INT &= PO \cdot r = R \frac{1 - (1 + r)^{-n+k-1}}{r} r \\ &= R \cdot (1 - (1 + r)^{-n+k-1}) \end{aligned}$$

Amount of kth payment Applied Towards Principal

Principal contained in the kth payment is the payment minus the above interest and is therefore given by the formula

$$\begin{aligned}
 PP &= R - INT \\
 &= R - R \cdot (1 - (1 + r)^{-n+k-1}) \\
 &= R(1 + r)^{-n+k-1}
 \end{aligned}$$

Total Interest Paid

The total interest paid is

$$TotInt = nR - A$$

Excel Functions for Calculating Loan Ammortizations

In the formulas below, the following arguments are used:

- Rate is the interest rate per period.
- Per is the period for which you want to find the interest and must be in the range 1 to nper.
- Nper is the total number of payment periods in an annuity.
- Pv is the present value, or the lump-sum amount that a series of future payments is worth right now.
- Fv is the future value, or a cash balance you want to attain after the last payment is made. If fv is omitted, it is assumed to be 0 (Which is the case for loans -- the future value of a loan is 0).
- Type is

0	Payments due at the end of the period (default if omitted).
1	Payments due at the beginning of the period

Interest payment for each period of a loan:

IPMT(rate,per,nper,pv,fv,type)

Returns the interest payment for a given period for an investment based on periodic, constant payments and a constant interest rate

The following formula calculates the interest due in the third month of a four-year \$6000 loan at 8 percent annual interest:

IPMT(0.08/12, 3, 48, 6000)

Payment at the end of each period of a loan:

PMT(rate, nper, pv, fv, type)

Returns the periodic payment for an annuity based on constant payments and a constant interest rate. Here we omit fv and type so that they default to 0.

Payment towards the principal at the end of each period of a loan:

PPMT(rate,per,nper,pv,fv,type)

Returns the payment on the principal for a given period for an investment based on periodic, constant payments and a constant interest rate. Here we omit fv and type so that they default to 0.