

Math As A Human Pursuit

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**How Probability Can be Used to Improve Your Odds  
in “Deal or No Deal” and The Monty Hall Problem**

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In our society, games of chance have constantly been created with different twists to stir our interests and emotions. For decades, games such as bingo, cards and dice were used to stimulate our need for chance. In all these types of games, mathematics has been used to determine odds and probability. When television was invented, these types of games grew in popularity. Game shows were soon created to center on contestants vying for random prizes, which ranged from cash to cars to trips around the world. The probability that the contestants would succeed was based on their knowledge and just pure chance.

One of today's most popular game shows is called "Deal or No Deal". There are two main versions of the show with one being played in the United Kingdom and the other here in the United States. Both have many of the same rules, but the United Kingdom has a slight twist in rules that will be discussed further in this paper. In this game show, there is a factor of pure chance, but mathematics can also be used in calculating the probability of a greater outcome.

Mathematical probability can be used to improve the odds in playing the game show, "Deal or No Deal" by determining the mean value of dollars left after each round when considering the banker's offer. The "Deal or No Deal" online game provides an excellent hands-on illustration of how the game works. Through manipulated probability calculations, one can determine if the probability of ending with a larger amount of money is greater than the offer from the bank. Recording calculations on a worksheet and comparing results help confirm the probability process and a sampling of data will reinforce this hypothesis (See handout).

The object of the game show, "Deal or No Deal", which originated in Holland and is now being broadcast in over 60 countries, is relatively simple and obviously popular (Forelle). The

show first aired in the United States in 2005, and is still showing today. The point of this game is to plainly assess if taking the 'Deal' is a better option than choosing 'No Deal'. The rules are simple and there is not much skill required; only the ability to make decisions is necessary. The game requires a contestant, a host, a banker and 26 cases each representing an amount of money from \$0.01 to \$1,000,000. At the beginning of the game, the contestant is presented with the 26 cases and chooses one of these cases to be set aside as his/hers throughout the game. The other 25 cases are then eliminated in a certain method in order for the contestant to get an idea of how much the case he/she originally chose is worth. (Deal or No Deal: Beat the Banker Game)

The contestant first chooses 6 cases to eliminate, and is shown the money value of each case eliminated. These amounts are then erased from the board because the contestant now knows that these particular money values are not in his/her case. The contestant is then given an offer from the banker. This offer is usually close to the average/mean of the amounts still available in the cases. The banker's offer is used to persuade the contestant to stop playing the game and to take the deal/proposed money. The contestant has the choice to say 'Deal' and take the money, or say 'No Deal' and continue playing in hopes of winning a larger amount of money later on in the game. Another offer is made after the next 5 cases are opened and eliminated, and again after the next sets of 4, 3, and 2 cases are opened (if the contestant chooses to take the 'No Deal' option presented). The remaining cases are opened one at a time, with a deal proposed after each one until the contestant decides that the amount in their original case is greater than any of the amounts still left in the remaining cases. In the end, the

contestant has the option to trade his/her original case with whichever remaining case is left, if they think the remaining case holds a larger amount of money. (Yahoo)

When examining the game, it does not seem like there would be much mathematics involved, however, there are actually two ways that mathematics are used in this game show. The first is found merely by looking at the probability of choosing one certain case. Because the contestant is choosing out of 26 cases, the probability of picking any certain one is calculated to be  $1/26$ . This probability is the same for any case chosen. Even when it comes down to two remaining cases, the probability of choosing one case over another is still  $1/26$ .

The second way that mathematics is applied to this game show occurs when deciphering if the deal presented by the banker is worth accepting or if the better option is to choose 'No Deal'. In order to decide which option is better, the average dollar (mean) amount remaining in the cases needs to be calculated. This is done by adding up all of the amounts that have not yet been eliminated and then dividing that total by the number of cases that are still remaining. If the amount that the banker offers the contestant is less than the mean or average, then the better decision is to continue on in the game and to not accept the deal. If the banker's amount is higher than the mean, then it would be statistically smarter to accept the deal and stop playing the game. If the contestant makes it their goal for the game to solely beat the mean amount then they have an advantage in their decision making and will have a better chance of coming away from this game with money in their pockets. (Deal or No Deal: A Statistical Deal)

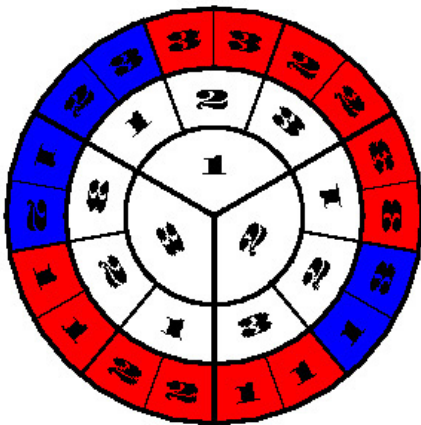
Another excellent and somewhat controversial game show that utilizes mathematical probability is “Let’s Make a Deal” and it is often compared/contrasted to the more modern “Deal or No Deal”. The concept of this Monty Hall Problem/Paradox so named after the host of the popular game show, “Let’s Make a Deal”, that aired in the 1970’s has created much discussion and can be found on many mathematical and probability websites.

An example of how this game works is as follows:

Suppose a contestant on a game show was given the choice of three doors. Behind one door is a car and behind the other two doors are goats. Say, the contestant picks Door #1 and Monty Hall, who knows what is behind each door, opens another door, for example, Door #2 that reveals one of the goats. Monty then offers the contestant an opportunity to change his or her selection to Door #3. Should the contestant stick with the original choice or switch doors? Does switching increase your chances of winning the car?

Assuming that Monty Hall always opens one of the doors with a goat and would never reveal the car, the assumption is that the contestant should switch to the Door #3, which is now twice as likely as the original door choice to be hiding the car. When the contestant chose Door #1, the probability that he or she chose the car was  $1/3$  and the probability that it was behind one of the other doors was  $2/3$ . By showing the contestant which of the remaining doors does not hide the car, for example Door #2, Monty is giving the contestant quite a bit of information about those two doors. The probability is still  $2/3$  that one of them hides the car, but now you know which of the two it would be, in this scenario Door #3. So, the probability is still only  $1/3$  that the car is behind Door #1, but the probability is  $2/3$  that it is behind Door #2.

To analyze this problem further, we represent this scenario as a random variable on a number wheel. The number wheel below simulates the “Let's Make a Deal” game. The inner wheel represents the number of the door that the car is behind, the middle wheel represents the door that is selected by the contestant, and the outer wheel represents the door Monty Hall can reveal. Spinning this number wheel would represent playing the game once. The outer wheel also tells you what your strategy should be to win. The red means that in order to win the contestant needs to switch doors, and the blue means that the contestant should not switch doors. Notice that there are twice as many red sections as blue. In other words, you are twice more likely to win if you switch than if you don't switch. What this wheel proves is that with a probability of 1/3 the contestant selects the correct door in which case it would be better not to switch. In the other 2/3 of the cases, Monty Hall is telling the contestant where the car is!



Chances by switching doors (Red areas) =

$$12/18 = 2/3$$

Chances by NOT switching doors (Blue areas)=

$$6/18=1/3$$

Another way to analyze this problem would be to make a Monty Hall Tree that would show all possible results, similar to our number wheel. This tree would actually show the odds of each outcome which again proves that the probability of winning the car by switching doors

is 2/3, while not switching doors has a probability of 1/3. Below is an example of a tree that we found on the internet.

Car location	Host opens	Car's probability	Stay	Switch
1/3 Door 1	1/2 Door 2	1/6	Car	Goat
	1/2 Door 3	1/6	Car	Goat
1/3 Door 2	1 Door 3	1/3	Goat	Car
	1/3 Door 1	1/6	Goat	Car

Notice that by adding the probabilities together you get the following results:

**Staying with original door (Not switching) =  $1/6 + 1/6 = 2/6 = 1/3$**   
**Switching doors =  $1/3 + 1/3 = 2/3$**

As mentioned earlier, we will now discuss whether the Monty Hall Problem would have any application to the show, "Deal or No Deal". First off, we would have to add the rule to the game that if the contestant gets down to the two final cases, then the host, Howie Mandel, would give the option of switching their original numbered case with the numbered case that is left on the board. The United Kingdom version of the show has this option, but the United States version does not. Second, we will assume that the two values left on the cash board are \$.01 and \$1,000,000. The question then becomes; should the contestant switch cases given the chance? In other words, is the probability higher that the \$1,000,000 is in the case that has not been picked, or is the chances equal for both cases?

The simple answer to this question is that the Monty Hall Problem cannot be used in this "Deal or No Deal" scenario that we set up. By looking at probabilities, we know that originally the contestant has a 1/26 chance of picking the case with the \$1,000,000 on their first

choice. From this we also know that after the original choice is chosen, there is a  $25/26$  chance that the \$1,000,000 is still left on the board. At this point, it seems obvious that switching cases would be the better choice for the contestant to make at the end. However, as the game plays on and each time the contestant chooses a case that is taken off the board, the odds then decrease by  $1/26$ . Therefore, once the game gets down to the last two cases, the probability for each case to contain the \$1,000,000 is  $1/26$ .

So in conclusion, there would be no advantage for the contestant to switch cases if given the opportunity in our “Deal or No Deal” scenario. One of the keys to the Monty Hall Problem is the fact that Monty knows which case contains the car in “Let’s Make a Deal”. If he had no prior knowledge of the winning door or always chose to open the same door number each time, then the probability of switching doors would change. In fact, if either of these scenarios occurred, then the probability of switching doors would actually be  $1/2$ .

As stated earlier, both of these game shows use probability mathematics in the decision-making process. Mathematics, however, is not the only factor that is utilized by contestants in games. Psychological factors may also play a role in why a contestant may be driven to choose one option over another. For example, contestants may use their intuition or previous life experiences as a way of gauging which option is the better choice. Economical factors, such as the amounts of money involved in a game, may also play a part in determining a person’s type of risk: risk acceptance or risk aversion.

From a psychological/emotional perspective, humans often look at individual decisions in the context of a set of needs, preferences and individual values being sought. They then

weigh whether their choice will bring a benefit or harm (Wikipedia). Cognition (mental processing) plays a part in this decision-making. According to behaviorist Isabel Briggs Myers, a person's decision making processes depend on their cognitive style. Per Briggs Myers, there are basically four main ways we experience or come to know the world: sensing, intuition, feeling, and thinking. (Kaplan and Saccuzzo 525).

But biases do exist. Some more commonly debated cognitive biases are (Wikipedia):

- listing the advantages and disadvantages of each option flipping a coin, cutting a deck of playing cards, and other random or coincidence methods
- accepting the first option that seems like it might achieve the desired result
- prayer, tarot cards, astrology, other forms of divination
- acquiesce to a person in authority or an "expert"
- calculating the expected value or utility for each option.
- believing we have more control over events than we really do.
- peer pressure to conform to the opinions held by others (a "Deal or No Deal" contestant even blamed her husband's "egging her on" as the reason she made a somewhat irrational choice. (Forelle 2006)

Some theories suggest that decisions are influenced by emotions, whether they are rational or irrational. In an article by Charles Forelle, Why Game Shows Have Economists Glued to Their TVs; For Researchers, Players Shed Light on Decision Making; Mr. Johnson's 'Gutsy' Move, a person's decision-making processes may at times be somewhat irrational. Contestants may deviate from rational behavior by being too conservative in their choices. The Classical economic theory states that people with "relatively small net worth, likely never again to see a large sum of money, would probably take a lower amount of money offered" (Wikipedia). An example of this would be a contestant accepting the banker's offer too early in "Deal or No Deal". Other times, people veer in a totally opposite direction and make irrationally risky decisions by not being conservative enough i.e., risk-accepting. A risk-accepting person

probably has suffered setbacks near the beginning of a game, such as opening the million-dollar briefcase too early (Wikipedia).

Additional factors that can cause a person to deviate from an otherwise structured approach to decision-making:

- Time restraint
- Higher stakes
- Increased ambiguities
- National or cross-cultural differences
- Our past circumstances even though they may no longer be relevant

We often make decisions without really thinking about what is going on behind the scenes. The following are some decision making techniques that we use in everyday life (Wikipedia):

- listing the advantages and disadvantages of each option, popularized by Plato and Benjamin Franklin
- flipping a coin, cutting a deck of playing cards, and other random or coincidence methods
- accepting the first option that seems like it might achieve the desired result
- prayer, tarot cards, astrology, augurs, revelation, or other forms of divination
- acquiesce to a person in authority or an "expert"
- calculating the expected value or utility for each option

So while some contestants will utilize the mathematical theories to give them an edge in formulating odds in games, others will still feel the need to rely on their more basic human nature and intuition to help them achieve that million dollar windfall.

Additional sites of interest:

- The Psychology of the Monty Hall Problem: Discovering Psychological Mechanisms for Solving a Tenacious Brain Teaser by Stefan Krauss
- Let's Make a Deal: Quality and availability of second-stage information as a catalyst for change by Jeffrey N. Howard, Charles G. Lambdin, and Darcee L. Datterri

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