

MTH111 Worksheet Confidence Intervals for Estimates of Population Proportions

Name _____

Definition: A c -confidence interval for the population proportion p is

$\hat{p} - E < p < \hat{p} + E$ where $E = z_c \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. Here z_c is the z score corresponding to c % and

$\hat{p} = \frac{x}{n}$ is the sample proportion, where n is the sample size and x is the number of positive responses, For a 95% confidence interval, the z -score is 1.96.

Steps for determining c confidence interval for opinion polls (or other proportion estimates). (Usual value for c is 95%).

1. Determine the total size of the sample n (which must be at least 30).
2. Determine the x , number in the sample that correspond to "Yes"
3. Compute $\hat{p} = \frac{x}{n}$, the sample proportion.
4. Find the critical value z_c -- the z - score corresponding to the given level of confidence c from the normal table.
5. Compute the margin of error as

$$E = z_c \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

6. Form the confidence interval : $\hat{p} - E < p < \hat{p} + E$

Exercise 1. Poll Margin of Error:

500 people surveyed. 52% plan to vote for Smith. Compute the 95% and 90% confidence intervals for this poll.

Exercise 2. A study of 1907 fatal traffic accidents found 449 of the fatalities were alcohol related. Construct the 95% and 99 % confidence intervals for this study.

Exercise 3: In a survey of 2001 adults, 461 would prefer to have a girl if they could only have one child. Construct the 95% and 99 % confidence intervals for this study.

Determining Sample Size Needed for Statistical Study to Estimate Population Proportions

To determine how large a sample size is needed for a desired confidence interval and margin of error:

1. If you have an estimated value for \hat{p} , use the formula $n \geq \hat{p}(1 - \hat{p})\left(\frac{z_c}{E}\right)^2$ where z_c is the maximum error of estimate and z_c is the critical z-score for the desired confidence interval.
2. If you do not have an estimated value for \hat{p} , use the formula $n \geq .25\left(\frac{z_c}{E}\right)^2$ where z_c is the maximum error of estimate and z_c is the critical z-score for the desired confidence interval.

Exercise 4: Suppose we are conducting a poll, and would like a 2% margin of error for a 95% confidence interval. What is the needed minimum sample size?

Exercise 5: A USA Today poll after the liberation of Kuwait reported that 91% of the respondents approved of George Bush's performance as president, with a margin of error of 4%. (Assuming a confidence interval of 95%). What was the sample size used in the poll?

Exercise 6: You wish to estimate with 96% confidence the proportion of camcorders that have problems by the time the product is five years old. Your estimate must be accurate to within 2.5% (plus or minus) of the true proportion.

a) Find the minimum sample size needed, assuming no prior estimate for the proportion.

b) Find the minimum sample size needed if you have a preliminary estimate that about 25% of the products need repairs (based on older data).