Functions

- A function describes how a **dependent variable** changes *with respect to* one or more **independent variables.** When there are only two variables, we often summarize them as an ordered pair with the independent variable first:
  (*independent variable, dependent variable)*
- We say that the dependent variable is a *function of* the independent variable. If x is the independent variable and y is the dependent variable, we write the function as $y = f(x)$

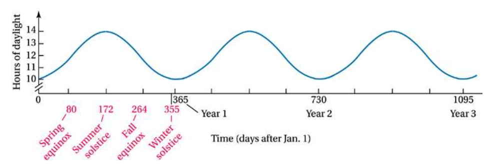
Representing Functions

- There are three basic ways to represent functions.
  - Data Table
  - Draw a *picture* or graph
  - Write an equation

Domain and Range

- The **domain** of a function is the set of values that both make sense and are of interest for the independent variable.
- The **range** of a function consists of the values of the dependent variable that correspond to the values in the domain.

Hours of Daylight as a Function



Linear Functions

Linear Functions

A **linear function** has a constant rate of change and a straight-line graph. For all linear functions,

- The rate of change is equal to the slope of the graph.
- The greater the rate of change, the steeper the slope.
- We can calculate the rate of change by finding the slope between any two points on the graph (Figure 9.9):

$$\text{rate of change} = \text{slope} = \frac{\text{change in } \textit{dependent variable}}{\text{change in } \textit{independent variable}}$$

Slope-Intercept Form of a Line

$y = mx + b$

$y = 3x – 7$

refers to a line whose slope is equal to 3 and a y-intercept of 7.

Exponential Functions

Exponential Functions

An **exponential function** grows (or decays) by the same relative amount per unit time. For any quantity Q growing exponentially with a fractional growth rate r,

$$Q = Q_0 \times (1 + r)^t$$

where

t = time
Q = value of the exponentially growing quantity at time t
Q_0 = initial value of the quantity (at $t = 0$)
r = fractional growth rate for the quantity

Negative values of r correspond to exponential decay. Note that *the units of time used for t and r must be the same.* For example, if the fractional growth rate is 0.05 per *month,* then t must also be measured in months.

Two Ways to Describe Exponentials

Forms of the Exponential Function

- If given the growth or decay rate r, use the exponential function in the form

$$Q = Q_0 \times (1 + r)^t$$

  Remember that r is positive for growth and negative for decay.
- If given the doubling time T_{double}, use the exponential function in the form

$$Q = Q_0 \times 2^{t/T_{double}}$$

- If given the half-life T_{half}, use the exponential function in the form

$$Q = Q_0 \times \left(\frac{1}{2}\right)^{t/T_{half}}$$

Changing Rates of Change