

Unit 3A

Uses and Abuses of Percentages

Three Ways of Using Percentages

- As fractions:

15% of the of the 850 students in a school were absent.

- To describe change:

The price of a stock increased 75% from \$50 per share.

- For comparisons:

A Mercedes costs 25% more than a Lexus.

Absolute and Relative Change

- The **absolute change** describes the actual increase or decrease from a **reference value** (starting number) to a **new value**:

$$\text{absolute change} = \text{new value} - \text{reference value}$$

- The **relative change** is a fraction that describes the size of the absolute change in comparison to the **reference value**:

$$\text{relative change} = \frac{\text{absolute change}}{\text{reference value}} = \frac{\text{new value} - \text{reference value}}{\text{reference value}}$$

Absolute vs. Relative Change

Example: A diversified portfolio grows from \$1,500 to \$2,250.

$$\begin{aligned}\text{absolute change} &= \text{new value} - \text{reference value} \\ &= \$2,250 - \$1,500 = \$750\end{aligned}$$

$$\begin{aligned}\text{relative change} &= \frac{\text{absolute change}}{\text{reference value}} \\ &= \$750 / \$1,500 = 0.5 = 50\%\end{aligned}$$

Absolute and Relative Difference

- The **absolute difference** is the actual difference between the compared value and the reference value:

absolute difference = compared value – reference value

- The **relative difference** describes the size of the absolute difference as a fraction of the reference value:

$$\begin{aligned}\text{relative difference} &= \frac{\text{absolutedifference}}{\text{referencevalue}} \\ &= \frac{\text{comparedvalue} - \text{referencevalue}}{\text{referencevalue}}\end{aligned}$$

Of versus More Than (or Less Than)

- If the compared value is *P% more than* the reference value, it is **(100 + P)%** *of* the reference value.
- If the compared value is *P% less than* the reference value, it is **(100 + P)%** *of* the reference value.

Percentages of Percentages

When a change or difference is expressed

- in **percentage points**, assume it is an **absolute** change or difference.
- with the % sign or the word *percent*, it is a **relative** change or difference.

Example: If a bank increases its interest rate from 4% to 5%, the interest rate increased by **1 percentage point**.

$$\text{relative change} = \frac{5\% - 4\%}{4\%} = 0.25 = 25\%$$

Solving Percentage Problems

- If the compared value is *P% more than* the reference value, then

$$\text{compared value} = (100 + P)\% \times \text{reference value}$$

and

$$\text{reference value} = \frac{\text{compared value}}{(100 + P)\%}$$

- If the compared value is *less than* the reference value, use $(100 - P)$ instead of $(100 + P)$ in the above calculations.

Solving Percentage Problems

You purchase a shirt with a labeled (pre-tax) price of \$21. The local sales tax rate is 6%. What is your final cost (including tax)?

$$\begin{aligned}\text{final cost} &= \text{labeled price} + (6\% \text{ of labeled price}) \\ &= (100 + 6)\% \times \text{labeled price} \\ &= 106\% \times \$21 = 1.06 \times \$21 = \$22.26\end{aligned}$$

Abuses of Percentages

- Beware of Shifting Reference Values

A 10% pay cut is followed by a 10% pay raise.

- Less than Nothing

Decrease caloric intake by 150% to lose weight.

- Don't Average Percentages

If 70% of the boys and 60% of the girls in a class voted to go to a water park, then 65% of the students in the class voted to go to the water park.