

Music Scales and Mathematics

TABLE 10.1
Frequencies of Notes in the Octave Above Middle C

NOTE	FREQUENCY (CPS)	RATIO TO FREQUENCY OF PRECEDING NOTE	RATIO TO FREQUENCY OF MIDDLE C
C	260	$\sqrt[12]{2} \approx 1.05946$	1.00000 = 1
C#	275	$\sqrt[12]{2} \approx 1.05946$	1.05946
D (second)	292	$\sqrt[12]{2} \approx 1.05946$	1.12246
D#	309	$\sqrt[12]{2} \approx 1.05946$	1.18921
E (third)	328	$\sqrt[12]{2} \approx 1.05946$	1.25992 $\approx \frac{5}{4}$
F (fourth)	347	$\sqrt[12]{2} \approx 1.05946$	1.33484 $\approx \frac{4}{3}$
F#	368	$\sqrt[12]{2} \approx 1.05946$	1.41421
G (fifth)	390	$\sqrt[12]{2} \approx 1.05946$	1.49831 $\approx \frac{3}{2}$
G#	413	$\sqrt[12]{2} \approx 1.05946$	1.58740
A (sixth)	437	$\sqrt[12]{2} \approx 1.05946$	1.68179 $\approx \frac{5}{3}$
A#	463	$\sqrt[12]{2} \approx 1.05946$	1.78180
B (seventh)	491	$\sqrt[12]{2} \approx 1.05946$	1.88775
C (octave)	520	$\sqrt[12]{2} \approx 1.05946$	2.00000 = 2

A Curious Golden Ratio Limit

Nested Square Roots of 1

$$\sqrt{1 + \sqrt{1}}$$

Value

1.41421356237 . . .

A Curious Golden Ratio Limit

Nested Square Roots of 1

$$\sqrt{1 + \sqrt{1}}$$

1.41421356237 ...

$$\sqrt{1 + \sqrt{1 + \sqrt{1}}}$$

1.55377397403 ...

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Nested Square Roots of 1

$$\sqrt{1 + \sqrt{1}}$$

1.41421356237 ...

$$\sqrt{1 + \sqrt{1 + \sqrt{1}}}$$

1.55377397403 ...

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}}$$

1.59805318248 ...

A Curious Golden Ratio Limit

Nested Square Roots of 1

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$$\sqrt{1 + \sqrt{1 + \sqrt{1}}}$$

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$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}}$$

1.59805318248 ...

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}}}$$

1.61184775413 ...

A Curious Golden Ratio Limit

Nested Square Roots of 1

Value

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1.41421356237 ...

$$\sqrt{1 + \sqrt{1 + \sqrt{1}}}$$

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$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}}$$

1.59805318248 ...

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}}}$$

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$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}}}}$$

1.61612120651 ...

A Curious Golden Ratio Limit

Nested Square Roots of 1

Value

$$\sqrt{1 + \sqrt{1}}$$

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$$\sqrt{1 + \sqrt{1 + \sqrt{1}}}$$

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1.59805318248 ...

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}}}$$

1.61184775413 ...

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}}}}$$

1.61612120651 ...

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}}}$$

$$\frac{1 + \sqrt{5}}{2} = \phi$$

As you continue to nest more roots, you will get infinitely closer to the golden ratio.

Parthenon Proportions & Golden Ratio

11-C

