How Probability Can Win You Money!

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Fundamentals of Probability

Outcomes

- The most basic possible results of observations or experiments

Events

- Consists of one or more outcomes that share a property of interest

Event vs. Outcome

- There are 4 different ways the coins can fall, which is known as the 4 outcomes.
- If we just want to know the number of heads, these are the events of the experiment.

Outcome				
Event	2 heads	1 head	1 head	0 heads

Examples

- Suppose you toss one coin three times in a row and get heads, tails, heads (HTH). If you are interested in the number of heads that appear, which one of the following sets of three tosses has a different outcome but corresponds to the same event as the first set of tosses?
- a) THT b) HHT c) HHH
- Answer: b

One More

- Does it Make Sense?
  - When I toss four coins, there are four different outcomes that all represent the event of one head and three tails.
  - Answer: Yes!!
    - Outcomes for 4 Coins: HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, **HTTT**, THHH, THHT, THTH, **THTT**, TTHH, **TTHT**, **TTTH**, TTTT

Expressing Probability

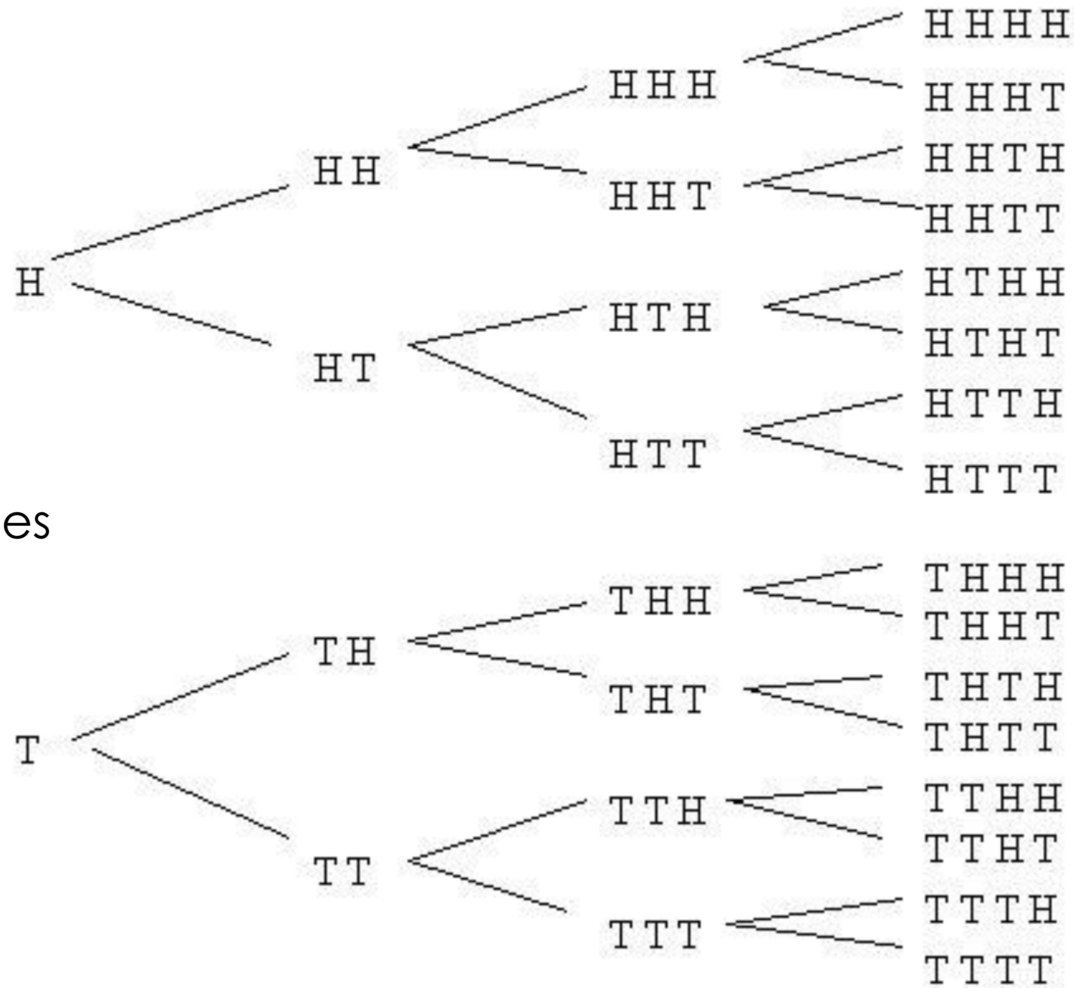
Visually

- Tree Diagrams
  - Expresses all the possible outcomes
  - Allows them to be easily read

Mathematically

- Numbers between 0 and 1
- 0 = impossible
- 1 = certain
- $P(A) = \frac{\text{number of ways } A \text{ can occur}}{\text{total number of outcomes}}$

Visual



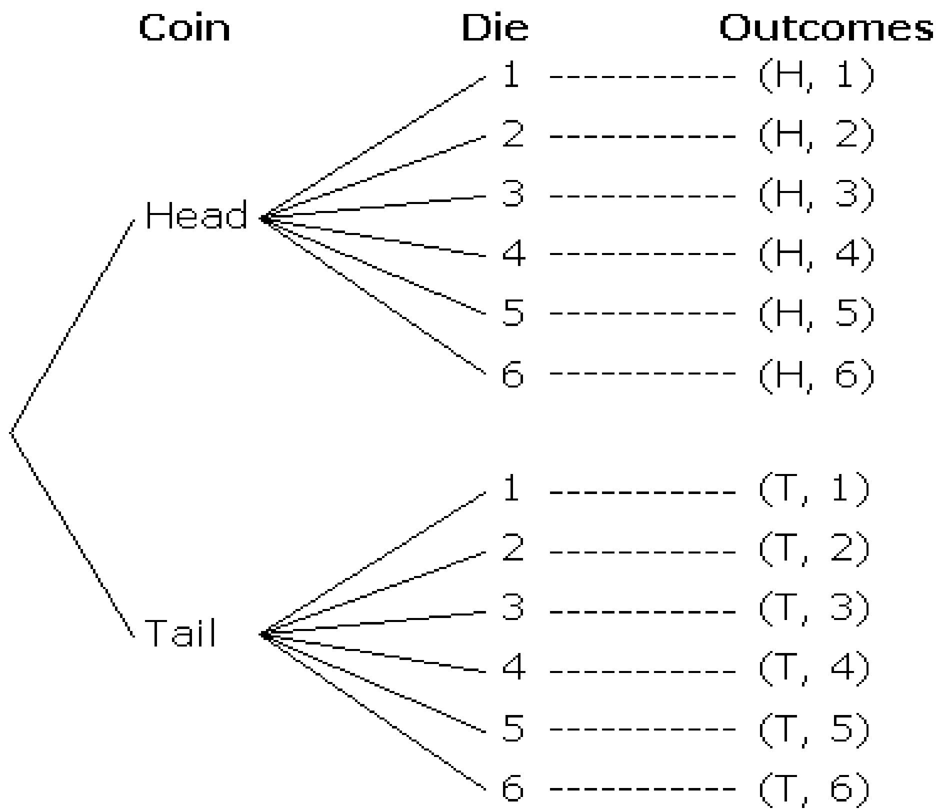
Tree Diagram
For the outcomes
Of 4 coin toss

Mathematically

- Probability of getting a head (H) when tossing a coin
  - $P(H) = \frac{1}{2} = 0.5$
- Probability of getting a black card in a standard deck of cards
  - $P(black\ card) = \frac{26}{52} = 0.5$
- Probability of getting a diamond in a standard deck of cards
  - $P(diamond) = \frac{13}{52} = 0.25$

Examples

- Visually express the outcomes of rolling a die and flipping one coin.
- Answer:



Examples

- Looking at the previous tree diagram express the probability mathematically of getting:
  - One head and an even number
    - $P(H, even\ \#) = \frac{3}{12} = 0.25$
  - A head or tail and an odd number
    - $P(H\ or\ T, odd\ \#) = \frac{6}{12} = 0.5$

Theoretical Probability

- Based on a model in which all outcomes are equally likely
- Theoretical Probability deals with fairness of the object being used, such as, coins and dice
- For example, the probability of getting a heads in a coin toss is ½. This probability is based on theory, and therefore is theoretical.
- Another example, the probability of rolling a 3 on a die is 1/6 which is also theoretical.

Theoretical Probability

- Method for Equally Likely Outcomes
  - Step 1: Count the total number of possible outcomes
  - Step 2: Among all the possible outcomes, count the number of ways the event of interest, A, can occur.
  - Step 3: Determine the probability, P(A), from
    - $P(A) = \frac{number\ of\ ways\ A\ can\ occur}{total\ number\ of\ outcomes}$

Empirical Probability

- Based on observations or experiments. It is the relative frequency of the event of interest
- For example, we know that Rensselaer has had 5 + inches off snow 15 times out of the last 20 years so we can claim that there is a probability of 15/20 or 0.75 it could snow 5+ inches any given year
- The probability is based on data collected instead of the theory that things are equally likely

Theoretical vs. Empirical

- Which is which?
  - A) The chance of rolling a 9 on a 12 sided die is 1/12.
  - B) Based on data from the Department of Transportation, the chance of dying in an automobile accident during a one-year period is 1 in 7000.

  - Answer: A) Theoretical B)Empirical

Demonstration

- We have a bag of marbles, there are 10 in the bag, 4 green and 6 blue.
- Draw a tree diagram for the possibilities when pulling out 2 marbles (replacing the marble after the first draw).
  - This will give us the theoretical probability
- Based on this, what is the probability of getting 1 of each color?
- Now, we are going to do the experiment.
- Draw one marble. Replace it. Draw another one.
  - Repeat 5 times.
  - This will be our empirical probability
- Based on this data, what is probability of getting 1 of each color?

Play the Game

The Monty Hall Problem

- http://www.youtube.com/watch?v=cXqDIFUB7YU
- http://www.nytimes.com/2008/04/08/science/08monty.html?_r=1#
- http://www.youtube.com/watch?v=mhlc7peGlGg

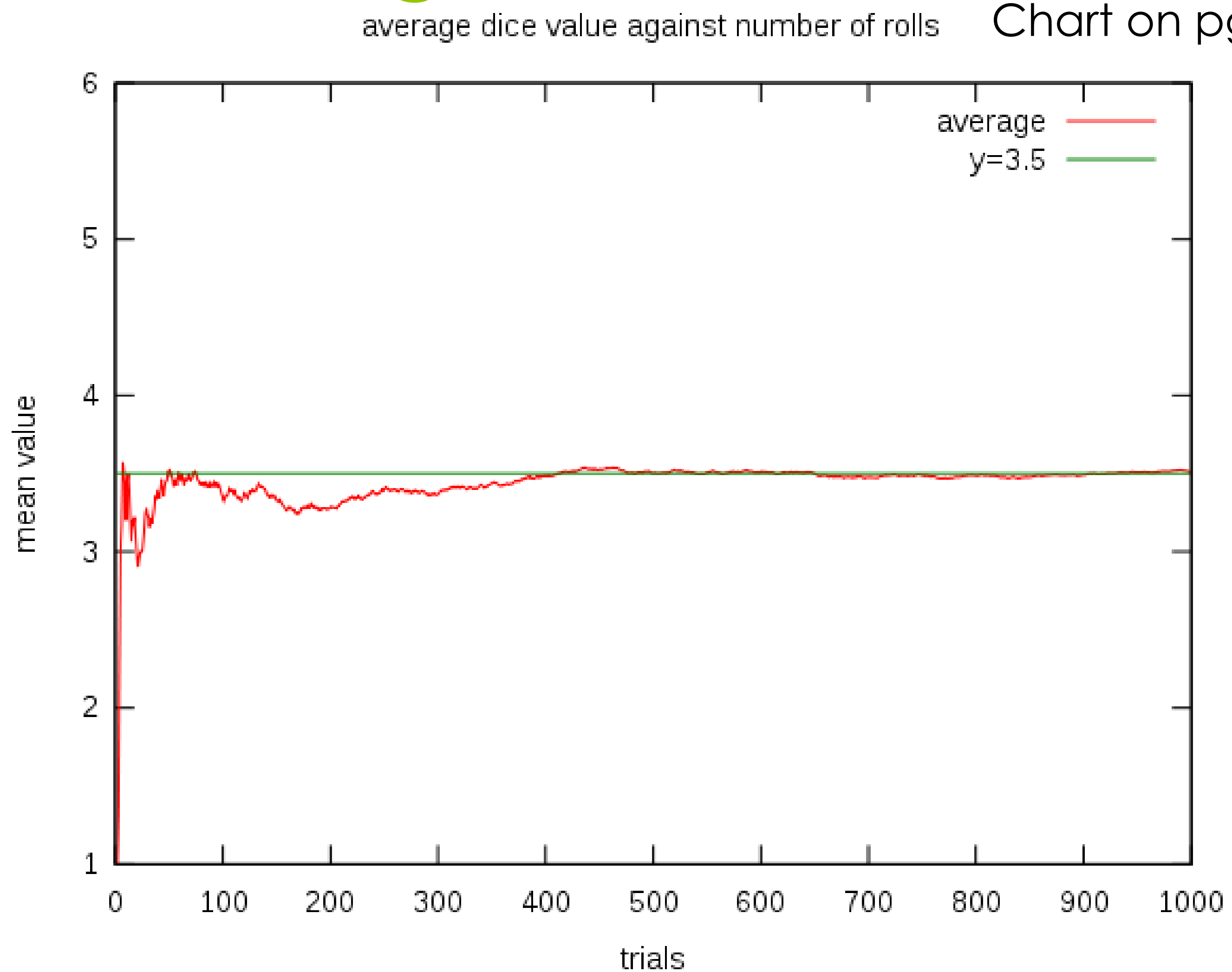
GAME TIME!!!

The Law of Large Numbers

- The law of large numbers applies to a process for which the probability of an event A is P(A) and the results of repeated trials are independent. It states: If the process is repeated over many trials, the proportion of the trials in which event A occurs will be close to the probability P(A). The larger the number of trials, the closer the proportion should be to P(A).

Law of Large Numbers

Chart on pg. 439



Roulette

- Named after a French diminutive for “little wheel”
- Players place bets on a number, a color, a range of numbers, or an odd or even number
- Whichever the ball lands on, the player wins.
- There are 38 numbers, 18 red, 18 black, and 2 green

Roulette

- A roulette wheel has 38 numbers, 18 are black, 18 are red, and the number 0 and 00 are green.
  - a) what is the probability of getting a red number on any spin?
  - b) if patrons in a casino spin the wheel 100,000 times, how many times should you expect a red number?

  - Answers: a) $\frac{18}{38} \approx 0.474$; b) 47.4% of the time or 47,400 times

Expected Value

- Multiplying the value of each event by its probability and then adding gives the expected value
- Expected value = (event 1 value) X (event 1 probability) + (event 2 value) X (event 2 probability)
- This can be applied to any number of events
- A good example is the Lottery

Lottery Example

- Suppose that $1 lottery tickets have the following probabilities: 1 in 5 to win a free ticket (worth $1), 1 in 100 to win $5, 1 in 100,000 to win $1000, and 1 in 10 million to win $ 1 million. What is the expected value of a lottery ticket?

Answer

Event	Value	Probability	Value x Probability
Ticket Purchased	-$1	1	-$1x1 = -$1
Win free ticket	$1	1/5	$1 x 1/5 = $0.20
Win $ 5	$5	1/100	$5 x 1/100 = $0.05
Win $ 1000	$1000	1/100,000	$1000 x 1/100,000 = $0.01
Win $ 1 million	$1,000,000	1/10,000,000	$1,000,000 x 1/10,000,000 = $0.10
			Sum = -$0.64

Averaged over many tickets, you should expect to lose $0.64 for each $ 1 ticket you buy. If you buy 1000 tickets, you should expect to lose about 1000 x $0.64 = $640.

House Edge

- The amount that the house, or casino, can expect to earn per dollar bet
- It varies among games
  - bigger among games where big winnings are possible and more chance involved (slot machines)
  - Smaller among games with more strategy to improve your odds (blackjack)
- Use the same idea for expected value

House Edge Example

- The game of roulette is usually set up so that betting on red is a 1 to 1 bet. That is, you win the same amount of money as you bet if red comes up. Betting on a single number is a 35 to 1 bet. That is, you win 35 times as much as you bet if your number comes up. What is the house edge in each of these two cases? If patrons wager $1 million on such bet, how much should the casino expect to earn?

Answer

- For red: Probability of getting red is 18/38 so the probability the house will win is 20/38.
  - So (\$1 x 20/38) + (-\$1 X 18/38) ≈ \$0.053
    - Casino can expect to gain \$0.053 per dollar gambled
    - Therefore in patron bets \$1 million, the casino can earn \$1 million x 0.053 = \$53,000
- For a single number: Probability of getting it is 1/38 so the probability the house will win is 37/38.
  - So (\$1 x 37/38) + (-\$35x 1/38) ≈ \$0.053
    - Casino can expect to gain \$0.053 per dollar gambled
    - Therefore in patron bets \$1 million, the casino can earn \$1 million x 0.053 = \$53,000

THE END!