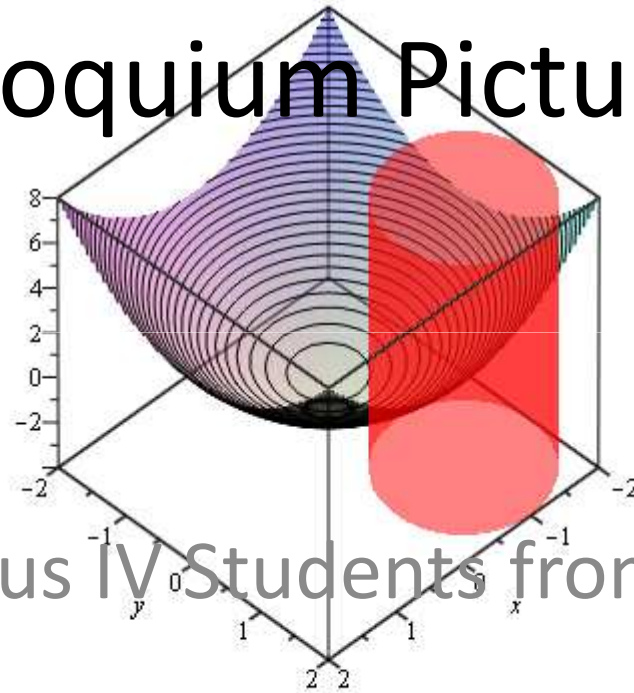


Saint Joseph's College Colloquium Pictures



Calculus IV Students from 2009

The DELTOID CURVE

Hypocycloids and the Deltoid Curve

Hypocycloids are a special class of the cycloids, where a point on a circle of fixed radius rolls on the inside of a larger circle. A special case is a trace of a fixed point on a small circle rolling within an outer larger circle. Here is an example.



The parametric equations for the deltoid are as follows:

$$\begin{aligned} x &= 2r \cos^3 t \\ y &= 2r \sin^3 t \end{aligned}$$

We make a deltoid by geometry and trigonometry

Let r be the radius of the circle that is rolling inside of a larger circle of radius R . The ratio of radii R/r and the angle θ are related by the following:

$$R/r = 1 + \cos \theta$$

Substituting in for R/r and simplifying the expression yields the following:

$$r = 2R \cos^2 \theta$$

However, since the new circle has radius r , we can get the radius of the circle rolling inside the larger circle as follows:

$$R = 2r \cos^2 \theta$$

From our picture, we can gather that a point on the smaller circle can be described with the following equation:

$$x = r \cos(\theta + \phi)$$

$$y = r \sin(\theta + \phi)$$

Using substitution we finally get the following:

$$x = 2r \cos^3 \theta$$

Example of a Deltoid

Imagine one has attached a string to a curve at a given point. This string is also tangent to the curve at that point. Say we wind the string around the curve while letting it stretch out as a tangent to every point, when we track out the endpoint of the string we get an involute. An example is shown below with a circle.



For hypocycloids, the involutes are unique because they are involutes of themselves. We can see this unique property by examining the picture of an involute below.



We can see that the "string" is projected as a tangent to a cusp. When we wind the string hypocycloid. For a deltoid, we obtain a smaller deltoid overlaid the size of our original.



Applications

Hypocycloids are used in a variety of fields. Hypocycloids are used in engineering for gear shapes and engine shaft design. Some gears follow deltoid-shaped curves. The deltoid planet inside of the human body is noted because it resembles the deltoid curve.



History of Hypocycloids and the Deltoid Curve

Deltoid first really took a "formal" name when it was named in 1644 by Simon Stevin. The name "deltoid" was actually first used by the mathematician Simon Stevin in 1644 when he was trying to design gear teeth.



The first known Cartesian curve that was a hypocycloid was the astroid. The astroid was first named by the mathematician Simon Stevin in 1644 when he was trying to design gear teeth.

TER FOWL



All the birds have the same shape for their wings. You may see many in the world. Through the thick night you can see the stars. And when they are full they are like diamonds. And when they are full they are like diamonds. He has been seen to swim through the fountain. In the long way that I have seen it, it had the same shape.



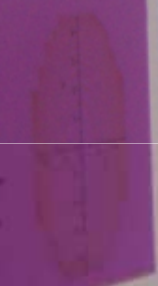




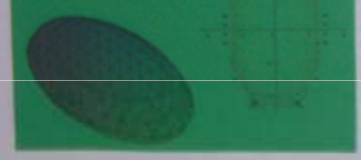
CONTOUR PLOTS & GRADIENT FIELDS

Contour Lines

Contour lines are lines on a map that connect points of equal elevation above a given reference elevation. They are drawn on a map to show the shape of the terrain. Contour lines are drawn at regular intervals, and the distance between two contour lines is called the contour interval. Contour lines are drawn in a way that they are perpendicular to the slope of the terrain. Contour lines are drawn in a way that they are parallel to each other. Contour lines are drawn in a way that they are closed loops. Contour lines are drawn in a way that they are not closed loops. Contour lines are drawn in a way that they are not closed loops. Contour lines are drawn in a way that they are not closed loops.



Ellipsoid

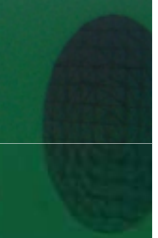


Hyperboloid of One Sheet



Gradient Fields

The gradient of a scalar field is a vector field that points in the direction of the greatest increase of the scalar field. It is denoted by the symbol ∇ . The gradient of a scalar field is a vector field that points in the direction of the greatest increase of the scalar field. It is denoted by the symbol ∇ . The gradient of a scalar field is a vector field that points in the direction of the greatest increase of the scalar field. It is denoted by the symbol ∇ .



Graphing a Contour Map

Graphing a contour map involves plotting the contour lines of a function on a 2D plane. The contour lines are drawn at regular intervals, and the distance between two contour lines is called the contour interval. Contour lines are drawn in a way that they are perpendicular to the slope of the terrain. Contour lines are drawn in a way that they are parallel to each other. Contour lines are drawn in a way that they are closed loops. Contour lines are drawn in a way that they are not closed loops. Contour lines are drawn in a way that they are not closed loops.



Elliptic Cone



Hyperboloid of Two Sheets



Hyperbolic Paraboloid



Elliptic Paraboloid



Sketching a Gradient Field

Sketching a gradient field involves plotting the vector field of a scalar field on a 2D plane. The vector field is drawn at regular intervals, and the distance between two vector fields is called the contour interval. Vector fields are drawn in a way that they are perpendicular to the slope of the terrain. Vector fields are drawn in a way that they are parallel to each other. Vector fields are drawn in a way that they are closed loops. Vector fields are drawn in a way that they are not closed loops. Vector fields are drawn in a way that they are not closed loops.







LAGRANGE MULTIPLIERS

$t_z = \lambda g_z$
 $y = 9x + 1$
 $2y + x^2 = 6$
 $(\sqrt{2}, 3)$

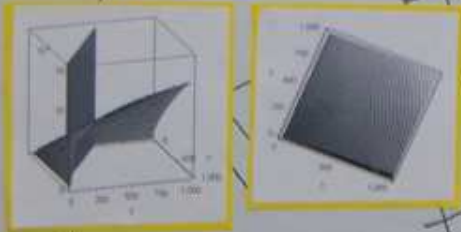
$f(x,y,z) = xyz$
 $\lambda \nabla f = \mu \nabla g + \nu \nabla h$
 $\frac{x}{3}$
 $f_y = \lambda g_y$

History:
 Joseph-Louis Lagrange (25 January 1736 – 17 April 1813) was an Italian mathematician and astronomer who made many contributions to all fields of science. His number theory and calculus work, including his work on the calculus of variations, is still important today. Lagrange's work on the calculus of variations led to the development of the calculus of variations, which is used in many areas of physics and engineering. He also made important contributions to the theory of algebra and the theory of numbers.

Problem:
 The production function for the company is given by $f(x,y,z) = 120x^{0.2}y^{0.3}z^{0.5}$. Adam's represents the units of labor at \$25 per unit and z represents the units of capital at \$120 per unit. The total cost of labor and capital is limited to \$48,000. Find the maximum production level.

Find within the domain by gradient:
 $f(x,y,z) = 120x^{0.2}y^{0.3}z^{0.5}$
 Need to find the constraint function:
 $g(x,y,z) = 25x + 120z = 48000$

Then plot the constraint function together and observe their intersection.



Applying the Method of Lagrange Multiplier. We use the setting $\lambda f(x,y,z) - g(x,y,z)$ to find the differentials:
 $\nabla \lambda f(x,y,z) - \nabla g(x,y,z) = 0$
 Differentiate f with respect to x and then with respect to y :
 $\lambda \frac{\partial f}{\partial x} = \frac{\partial g}{\partial x}$
 $\lambda \frac{\partial f}{\partial y} = \frac{\partial g}{\partial y}$

These values are the maximum values of f which occur on the constraint curve $g(x,y,z) = 0$. The Lagrange multiplier λ also can now be evaluated.
 The values of x and z which give the conditional maximum of $f(x,y,z)$ are found by solving these equations:
 $\frac{\partial f}{\partial x} = 0$
 $\frac{\partial f}{\partial y} = 0$
 $\frac{\partial f}{\partial z} = 0$

This is a process where the maximum value for this constrained problem. The maximum will occur where the level curve of $f(x,y,z)$ is tangent to the level curve $g(x,y,z) = 0$.

A contour plot of the function $f(x,y,z)$ is the plane along with the level curve $g(x,y,z) = 0$ show where the point at which the maximum occurs can be seen.



Check value of $f(x,y,z)$ at the point at which the maximum occurs. This point will be the maximum value of $f(x,y,z)$.

From the point (x,y,z) which gives the conditional maximum of $f(x,y,z)$ we find the level curve of $g(x,y,z)$ is tangent to the constraint $g(x,y,z) = 0$.



Lagrange's Theorem:
 Let $f(x,y,z)$ be a function of three variables. The point (x,y,z) is a local maximum of $f(x,y,z)$ if and only if there is a scalar λ such that $\lambda \nabla f(x,y,z) = 0$.



Conclusion:
 The maximum value of $f(x,y,z)$ is found at the point (x,y,z) where the level curve of $f(x,y,z)$ is tangent to the constraint $g(x,y,z) = 0$.

