Pythagorean Theorem

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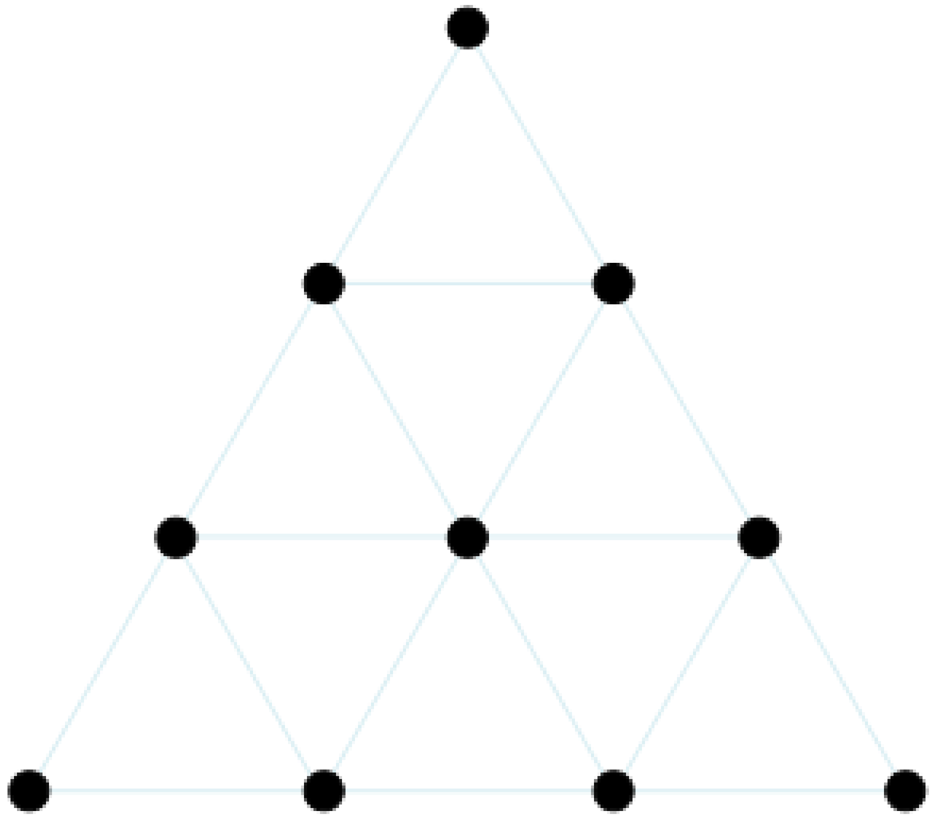
Pythagoras

- 570-495BC
- Greek Philosopher and Mathematician
- Turned music notes into equations
- Created Tetractys-Pythagoreans thought it was a mystic symbol

Tetractys

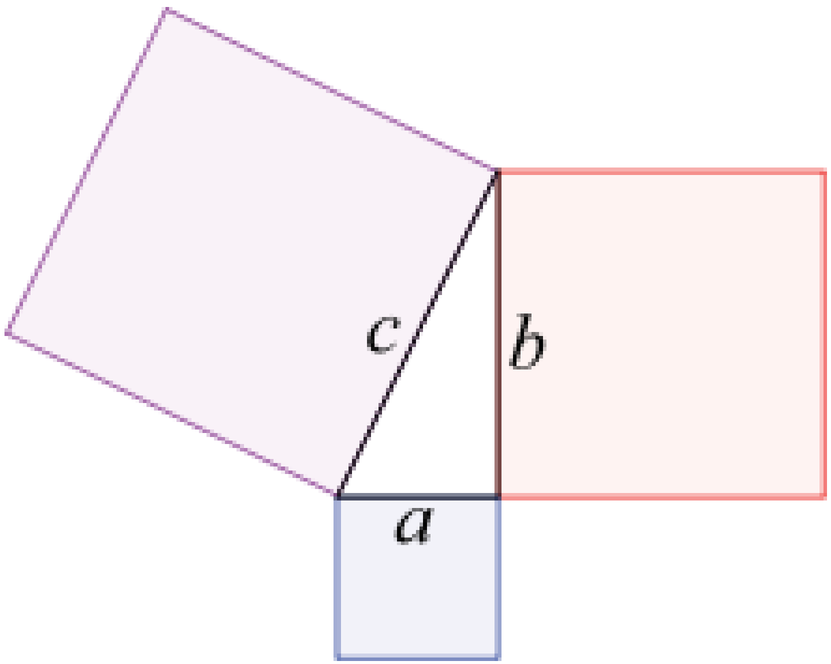
- The Tetractys symbolized the four elements — earth, air, fire, and water.
- The first four numbers also symbolized the harmony of the spheres and the Cosmos.
- The four rows added up to ten, which was unity of a higher order (in decimal).
- The Tetractys represented the organization of space:
  - the first row represented zero-dimensions (a point)
  - the second row represented one-dimension (a line of two points)
  - the third row represented two-dimensions (a plane defined by a triangle of three points)
  - the fourth row represented three-dimensions (a tetrahedron defined by four points)
- Pythagoreans- followers



Pythagorean Theorem

In any right triangle, the area of the square whose side is
the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares whose sides are the two legs (the two sides that meet at a right angle).

$$a^2 + b^2 = c^2$$



Other forms

$$a^2 + b^2 = c^2$$

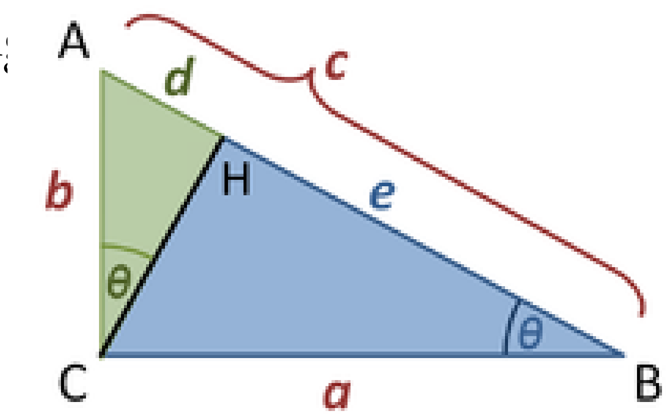
$$c = \sqrt{a^2 + b^2}.$$

$$a = \sqrt{c^2 - b^2}.$$

$$b = \sqrt{c^2 - a^2}.$$

Proof Using Similar Triangles

- Let *ABC* represent a right triangle, with the right angle located at *C*, as shown on the figure. We draw the altitude from point *C*, and call *H* its intersection with the side *AB*. Point *H* divides the length of the hypotenuse *c* into parts *d* and *e*. The new triangle *ACH* is similar to triangle *ABC*, because they both have a right angle (by definition of the altitude), and they share the angle at *A*, meaning that the third angle will be the same in both triangles as well, marked as θ in the figure. By a similar reasoning, the triangle *CBH* is also similar to *ABC*. The proof of similarity of the triangles requires the Triangle postulate: the sum of the angles in a trian



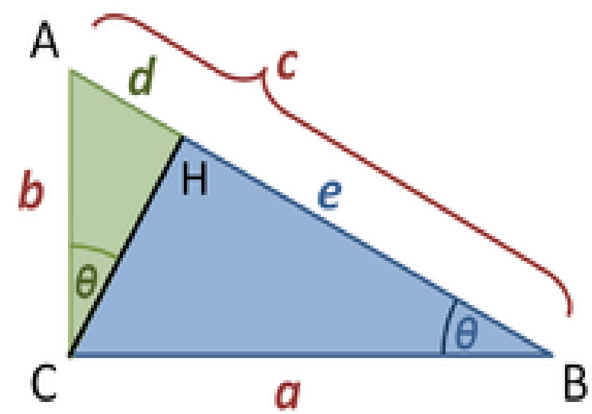
Proof Continued

- Similarity of the triangles leads to the equality of ratios of corresponding sides:

$$\frac{a}{c} = \frac{e}{a} \text{ and } \frac{b}{c} = \frac{d}{b}.$$

- The first result equates the cosine of each angle θ and the second result equates the sines.
- These ratios can be written as:

$$a^2 = c \times e \text{ and } b^2 = c \times d.$$



- Summing these two equalities, we obtain

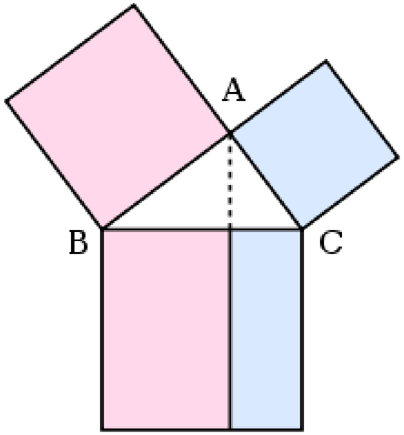
$$a^2 + b^2 = c \times e + c \times d = c \times (d+e) = c^2,$$

- which, tidying up, is the Pythagorean theorem:

$$a^2 + b^2 = c^2\ .$$

Proof Using Euclid

- The large square is divided into a left and right rectangle. A triangle is constructed that has half the area of the left rectangle. Then another triangle is constructed that has half the area of the square on the left-most side. These two triangles are shown to be congruent, proving this square has the same area as the left rectangle. This argument is followed by a similar version for the right rectangle and the remaining square. Putting the two rectangles together to reform the square on the hypotenuse, its area is the same as the sum of the area of the other two squares.

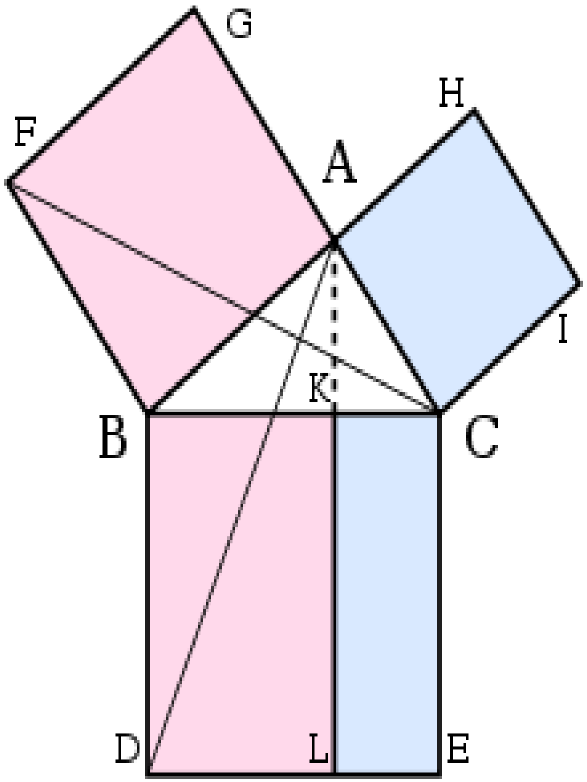


Euclid Proof Continued

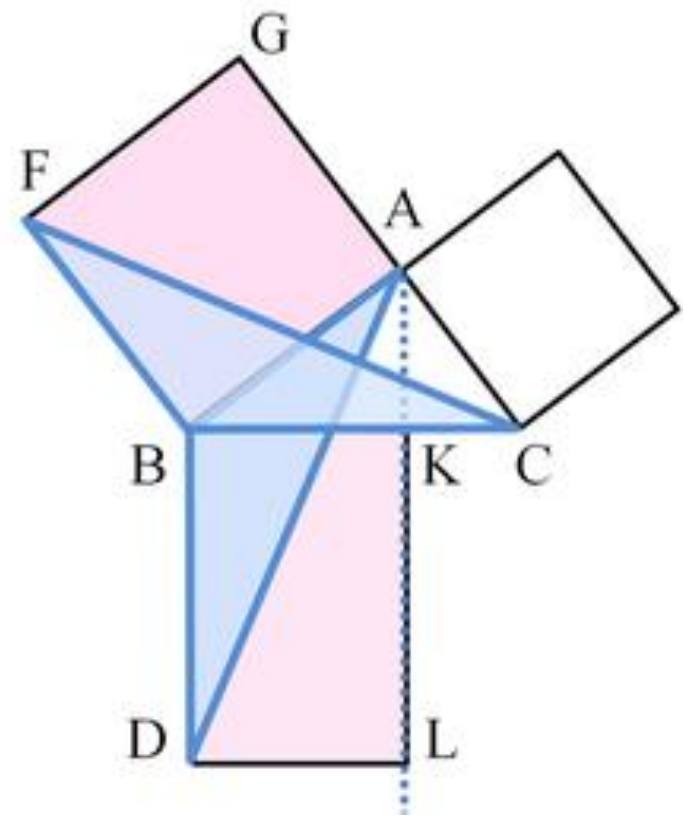
- Let *A*, *B*, *C* be the vertices of a right triangle, with a right angle at *A*. Drop a perpendicular from *A* to the side opposite the hypotenuse in the square on the hypotenuse. That line divides the square on the hypotenuse into two rectangles, each having the same area as one of the two squares on the legs.
- For the proof, we require four things that we assume to be valid:

1. If two triangles have two sides of the one equal to two sides of the other, each to each, and the angles included by those sides equal, then the triangles are congruent (side-angle-side).
2. The area of a triangle is half the area of any parallelogram on the same base and having the same altitude.
3. The area of a rectangle is equal to the product of two adjacent sides.
4. The area of a square is equal to the product of two of its sides (follows from 3).

- Let ACB be a right-angled triangle with right angle CAB.
- On each of the sides BC, AB, and CA, squares are drawn, CBDE, BAGF, and ACIH, in that order. The construction of squares requires the immediately preceding theorems in Euclid, and depends upon the parallel postulate.
- From A, draw a line parallel to BD and CE. It will perpendicularly intersect BC and DE at K and L, respectively.
- Join CF and AD, to form the triangles BCF and BDA.
- Angles CAB and BAG are both right angles; therefore C, A, and G are collinear. Similarly for B, A, and H.
- Showing the two congruent triangles of half the area of rectangle BDLK and square BAGF
- Angles CBD and FBA are both right angles; therefore angle ABD equals angle FBC, since both are the sum of a right angle and angle ABC.



- Since AB and BD are equal to FB and BC, respectively, triangle ABD must be congruent to triangle FBC.
- Since A is collinear with K and L, rectangle BDLK must be twice in area to triangle ABD, since it shares a height with BK and a base with BD and a triangle's area is half the product of its base and height.
- Since C is collinear with A and G, square BAGF must be twice in area to triangle FBC.
- Therefore rectangle BDLK must have the same area as square BAGF = AB^2.
- Similarly, it can be shown that rectangle CKLE must have the same area as square ACIH = AC^2.
- Adding these two results, $AB^2 + AC^2 = BD \times BK + KL \times KC$
- Since BD = KL, BD* BK + KL × KC = BD(BK + KC) = BD × BC
- Therefore $AB^2 + AC^2 = BC^2$, since CBDE is a square.



Proof by Rearrangement

- http://www.youtube.com/watch?v=o-i4l7rulhU

Pythagorean Triples

- Pythagorean triples are three positive integers a, b, c so that $a^2 + b^2 = c^2$

- EX. (3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (11, 60, 61), (12, 35, 37), (13, 84, 85), (16, 63, 65), (20, 21, 29), (28, 45, 53), (33, 56, 65), (36, 77, 85), (39, 80, 89), (48, 55, 73), (65, 72, 97)

Babylonian Triples?

- The Pythagorean Triples are credited to Pythagoras, but the Babylonians seemed to have an understanding about how these worked already

Plimpton 322 Tablet made by the Babylonians contained a list of Pythagorean triples

Conclusion

- http://www.youtube.com/watch?v=7qRSPEv316U