Pascal's Triangle and Binomial Expansions

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Blaise Pascal

- France
- 1623-1661
- 1655

The Triangle Basics

- The top begins with n = 0
- Rows add a new number starting with k = 0

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Triangle Construction

- Write a 1 at row 0
- Add numbers above (to the right and left)
- Add 0 if number is absent
- Any nonnegative integer n
- Any integer k between 0 and n

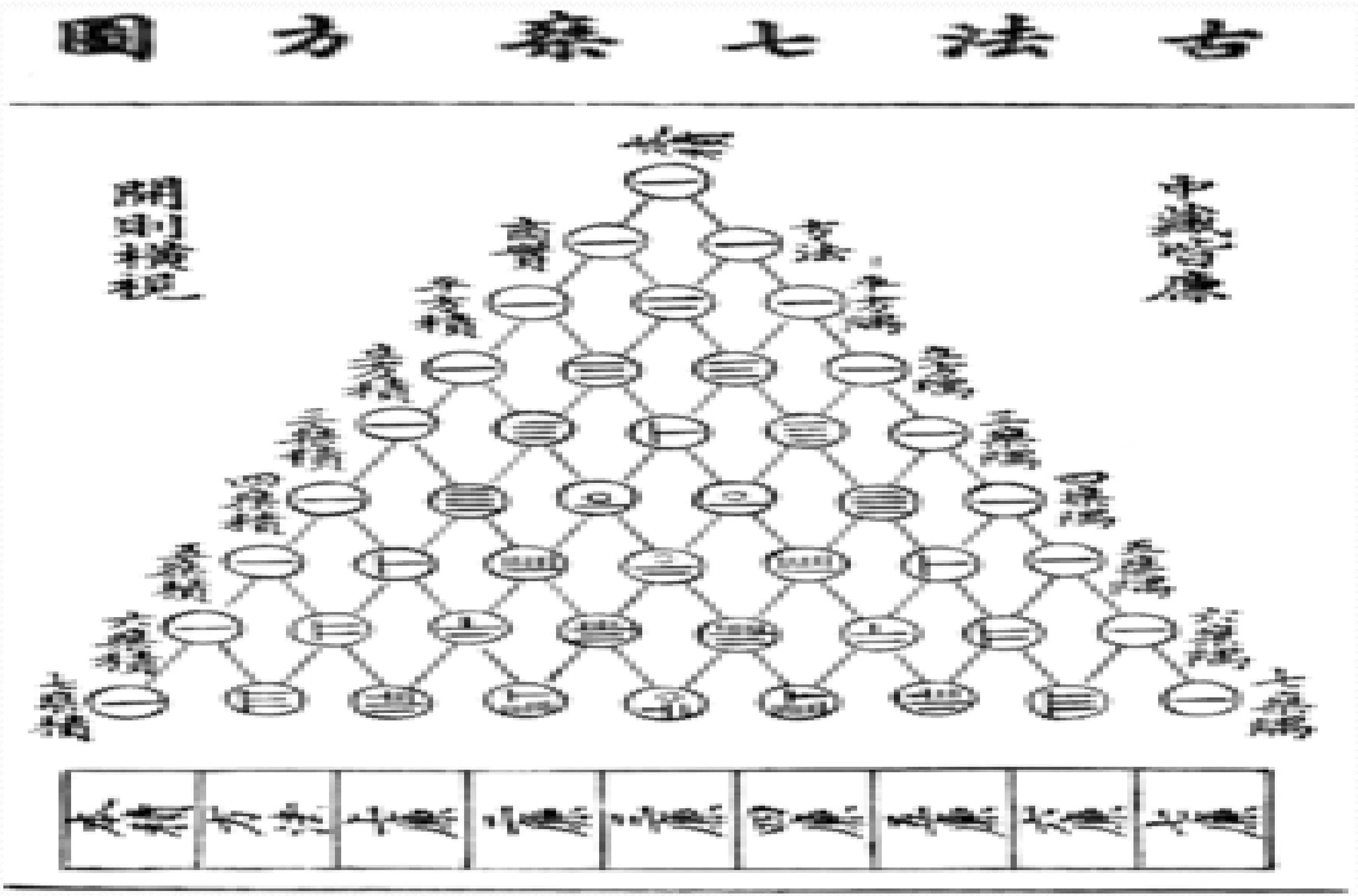
The Triangle

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

History

- Time before Pascal
- *Binomial Theorem*
- *Chandas Shastra 10th century India*
- *Pingala author*
- *Other work done in Greece, Persia, Iran, and China*
- *First to organize all the information*

Yang Hui’s Triangle



Binomial Expansions

$$(x + y)^2 = x^2 + 2xy + y^2 = \mathbf{1}x^2y^0 + \mathbf{2}x^1y^1 + \mathbf{1}x^0y^2$$

Row two: **1 2 1**

Binomials raised to a positive integer power:

$$(x + y)^n = a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + \ldots + a_{n-1}xy^{n-1} + a_ny^n$$

With row n, we get: a_i

Therefore, we get the Binomial Theorem:

$$a_i = \binom{n}{i}.$$

Binomial Theorem

- Far right diagonal parallels y^n

1. The next diagonal parrels xy^{n-1}

2. Proof: $(a + b)^n = b^n(a/b + 1)^n$

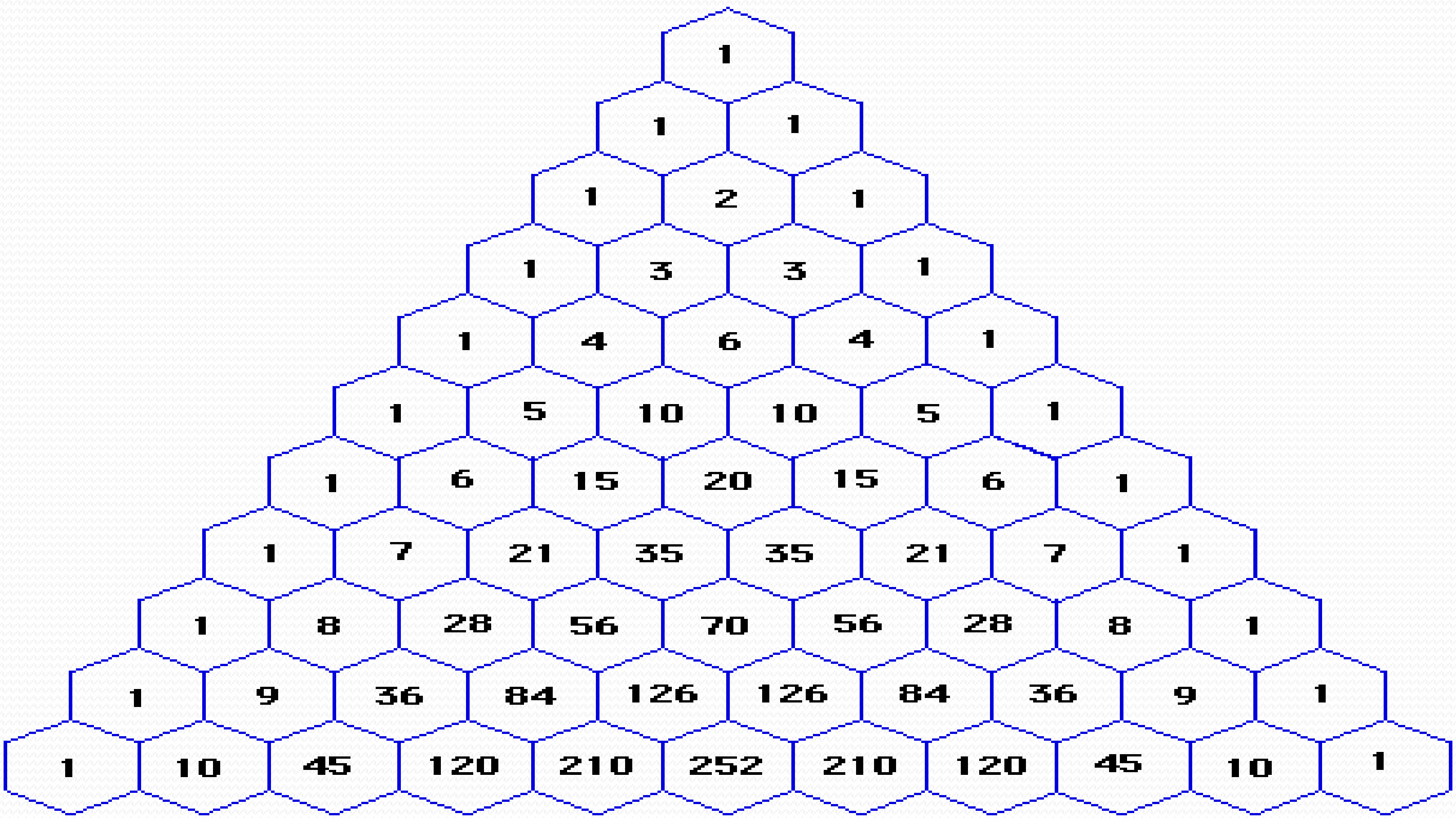
Combinations

- Triangle relevant to combinations
- Same formula used for both
- N choose k

$$C(n,k) = C_k^n = {}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Combination example

- 10 team members, how many ways to select 8
- 10 choose 8 = 45
- Remember, n = 0   k = 0



Conclusion

- Pascal’s contributions
- Prior work
- Useful organization
- Binomial Theorem