

ANCIENT GREEK FANTASIES ABOUT NUMBER

Arithmetic & Logistics

- ◉ Arithmetic: “a form of abstract wisdom”
- ◉ Logistical numerals regarded as quantities
- ◉ Experience came from the senses, not reason
- ◉ Parmenides: *reductio ad absurdum*
- ◉ An arrow must remain suspended in same place, forever
- ◉ Bring one in touch with ultimate reality → purpose of arithmetic

The Natural World

- What 'elements' make up the universe?
- Democritus: atomic theory, human senses
- Aristotle: biology research, dissection
- Plato: study of nature results in opinion, philosophical thinking gave knowledge
- Pre-Socratic works hardly exist

Greek Numbers

- Herodian/Attic numbers

- > Base 10
- > 5th_1st century AD
- > No zero

- Ionic/Attic numbers

- > 24 Greek letters
- > Too complicated

- Calculators

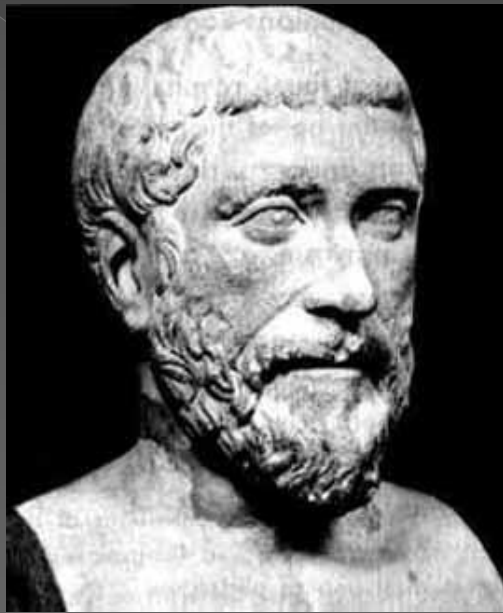
made with abacus
 tool, possibly with sand table



| | | |
|---------|---------|--------|
| H | X | M |
| Hekaton | Khilioi | Murioi |
| ἑκατόν | χίλιοι | μυριοί |
| 100 | 1000 | 10000 |

| | | | | | | | | |
|---|---------|-------|----|---------|--------|-----|---|-------|
| ι | iota | 100 | ρ | rho | | | | |
| κ | kappa | 200 | σ | sigma | | | | |
| λ | lambda | 300 | τ | tau | | | | |
| μ | mu | 400 | υ | upsilon | | | | |
| ν | nu | 500 | φ | phi | | | | |
| ξ | xi | 600 | χ | chi | | | | |
| ο | omicron | 700 | ψ | psi | | | | |
| 8 | η | eta | 80 | π | pi | 800 | ω | omega |
| 9 | θ | theta | 90 | Ϟ | koppa* | 900 | Ϸ | sampi |

*vau, koppa, and sampi are obsolete characters



Pythagoras

- Born in 580 BC, died in 500 BC
- Spent 20 years in Babylon
- 525 BC: secret society in Croton
- Credited with 2 discoveries
 - > Pythagorean Theorem
 - > Strings cause harmony or dissonance
- Theology of Number
 - > Good and evil
 - > Odd were female, even male
 - > Same gender could go together



Pythagoras' Secret Society

- ◉ The universe was created, and continues to exist, on the basis of a divine plan.
- ◉ God created souls as spiritual entities. The soul is a self-moving number which passes from body to body. Souls are eternal.
- ◉ There is an inner harmony and order in the universe. This results from the union of opposites. Ten fundamental opposites.
- ◉ In human relations, friendship and modesty are the most important principles.
- ◉ The divine ideas, which created and maintain the universe, are those of number.

Taxonomy of Number

- Definition of number
 - > Limited multitude
 - > A combination or heaping up of units
 - > A flow of quantity
- One and two are not numbers
- $1+2+3+4=10 \rightarrow$ Divine number, *tetraktos*
- Odd versus Even

| Even | 4, 6, 8, 10, 12 | Odd | 3, 5, 7, 9, 11 |
|-----------|--------------------|----------|-------------------|
| Even-even | 4, 8, 16, 32, 64 | Even-odd | 6, 10, 14, 18, 22 |
| Odd-even | 12, 20, 28, 36, 42 | Odd-odd | 9, 15, 21, 27, 33 |

- Lead to prime numbers and factors

- ◉ Creator's definite plan

- > $6=2 \times 3$; 1, 6 also factors
- > Ignore 6, because it is our original number
- > Also, $6=1+2+3 \rightarrow$ a 'perfect' number
- > Only four 'perfect' numbers: 6, 28, 496, 8128

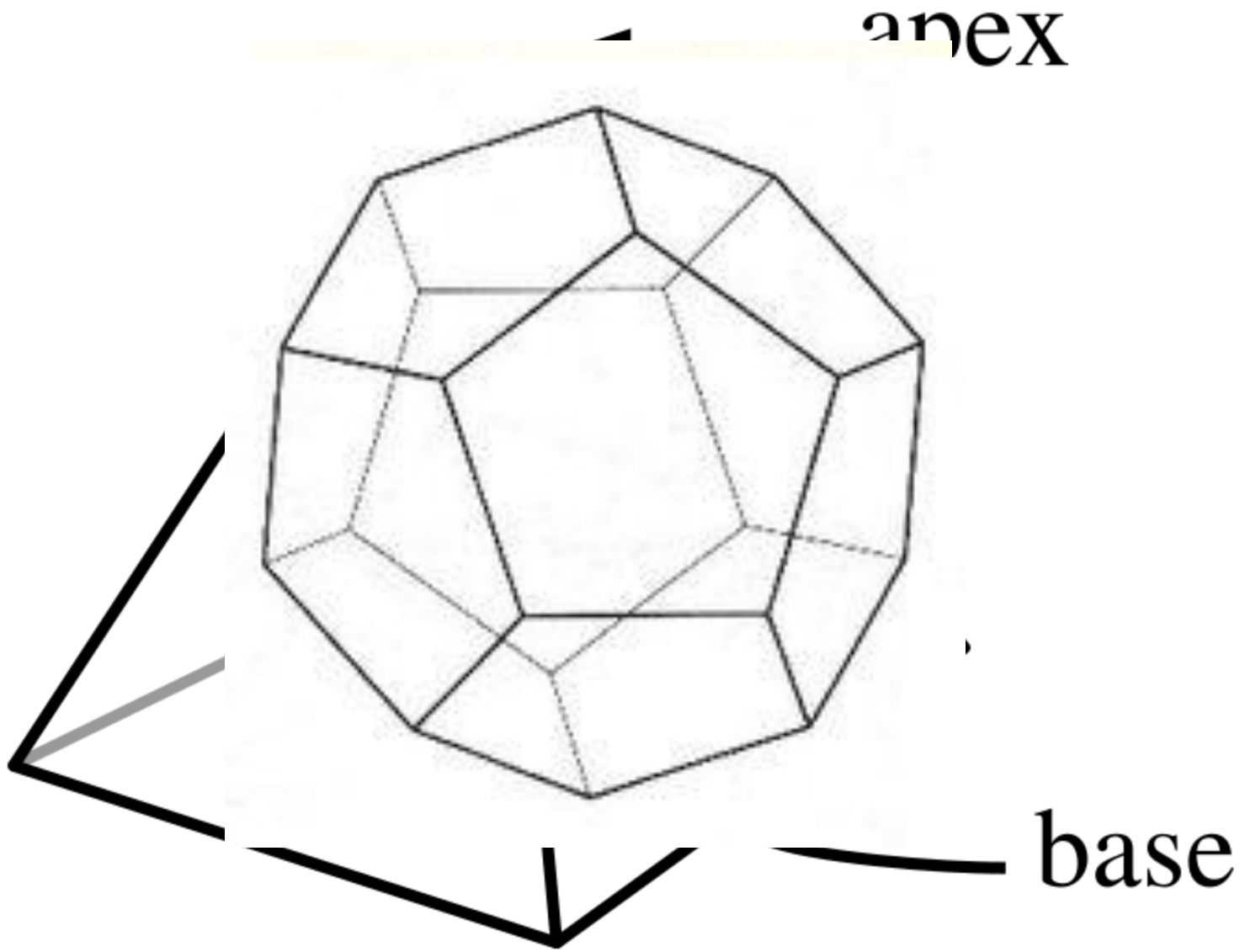
- ◉ Nicomachus: as with numbers, so with human qualities

- ◉ Amicable versus Unfriendly

- > 220: $1+2+4+5+10+11+20+22+44+55+110=284$
- > 284: $1+2+4+71+142=220$

Pythagoras & Geometry

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○ Born ~330 BC

○ Wrote *Elements*

> Defines parallel lines

> Propositions by deduction in a systematic order from stipulations

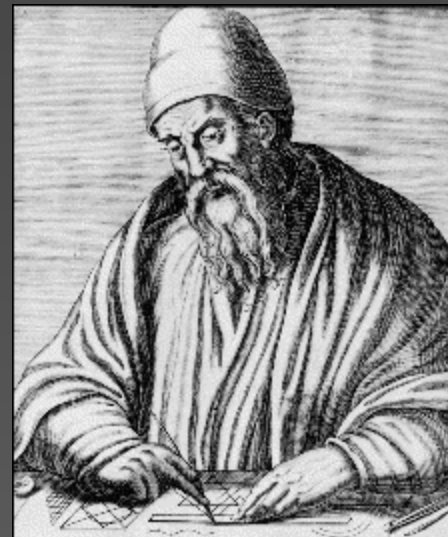
> No input from observation

> 'Flat-Earth' geometry

○ 1820s-1830s:

> Euclid's axioms

> Circular 'Earth' geometry



and

by deduction in the axioms and

merely deduction

ries proposed

Greek Algebra

- ◉ 'Rhetorical algebra' progresses to 'syncopated algebra'
- ◉ Syncopated algebra:
 - > A number of apples is divided among 6 people.
 - > 1st gets $\frac{1}{3}$, 2nd $\frac{1}{8}$, 3rd $\frac{1}{4}$, 4th $\frac{1}{5}$; The 5th gets 10 apples and 1 is left for the 6th.
 - > How many apples are there in total? 120
- ◉ Diophantus' *Arithmetika*
 - > Treats and solves algebraic problems
 - > Builds on 7th & 10th books of *Elements*
 - > Solves 189 problems

How Diophantus solved an algebra problem

i.e. If x^2 and y^2 are the two given square numbers, then $x^2y^2 + x^2$ and $x^2y^2 + y^2$ are also square numbers.

Diophantus gave the solution $x = \frac{7}{24}$ and $y = \frac{3}{4}$

There are, of course, an infinite number of solutions; but here is how Diophantus obtained his solution:

Solution

$x^2y^2 + x^2 = x^2(y^2 + 1)$, which is a square if $y^2 + 1$ is a square.

Suppose $y^2 + 1 = (y - 2)^2$

$$\Rightarrow y^2 + 1 = y^2 - 4y + 4$$

$$\Rightarrow 1 = -4y + 4$$

$$\Rightarrow y = \frac{3}{4}$$

But $x^2y^2 + y^2$ also has to be a square.

$$\Rightarrow \frac{9}{16}x^2 + \frac{9}{16} \text{ is a square}$$

$$\Rightarrow 9x^2 + 9 \text{ is a square}$$

$$\text{Assume } 9x^2 + 9 = (3x - 4)^2$$

$$\Rightarrow 9x^2 + 9 = 9x^2 - 24x + 16$$

$$\Rightarrow 9 = -24x + 16$$

$$\Rightarrow x = \frac{7}{24}$$

Thus the two numbers are $\frac{7}{24}$ and $\frac{3}{4}$

and the corresponding values of $x^2y^2 + x^2$ and $x^2y^2 + y^2$ are:

$$x^2y^2 + x^2 = \frac{1,225}{9,216} = \left(\frac{35}{96}\right)^2 \text{ and}$$

$$x^2y^2 + y^2 = \frac{5,625}{9,216} = \left(\frac{75}{96}\right)^2$$

Symbolic Algebra: the Greek contribution

Let the sum of the numbers be 20
and the sum of the squares be 208

Let the numbers be $10 + x$ and $10 - x$

Squaring gives $x^2 + 20x + 100$

$$x^2 - 20x + 100$$

Adding for sum of squares gives $2x^2 + 200 = 208$

Subtracting 200 from both sides gives $2x^2 = 8$

This must mean that $x^2 = 4$

And so the solution is that $x = 2$

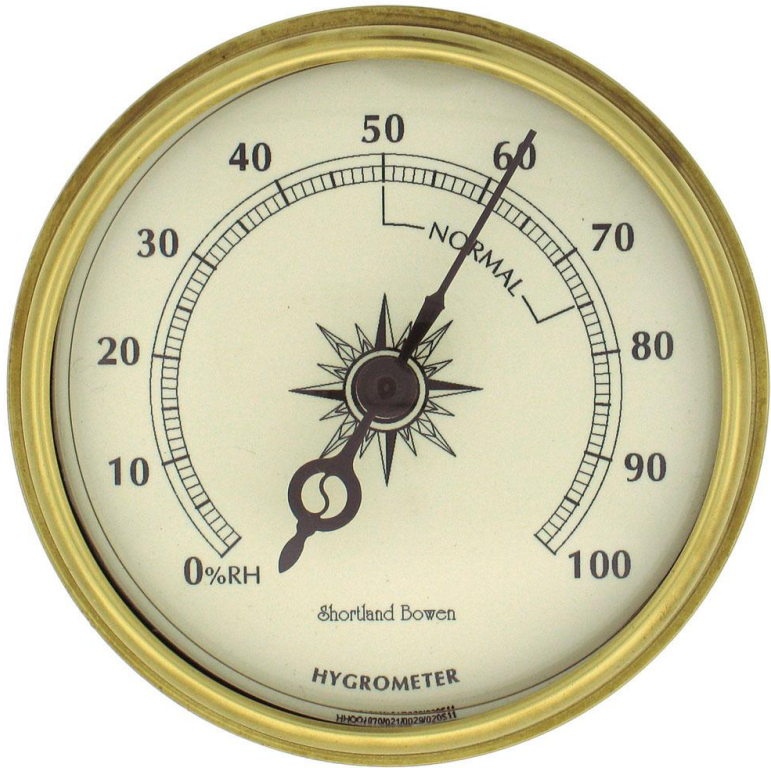
Hypatia (of Alexandria)



- ◉ Daughter and student of Theon, a Pythagorean who was a university professor
- ◉ Became professor of math and philosophy at the same school
- ◉ Pagan, Platonist, Pythagorean
- ◉ Caught in feud between Cyril & Orestes
- ◉ Dragged into a church, stripped and murdered, quartered and burned in 415
- ◉ Coincided with Dark Ages

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