Least Squares

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Least Squares

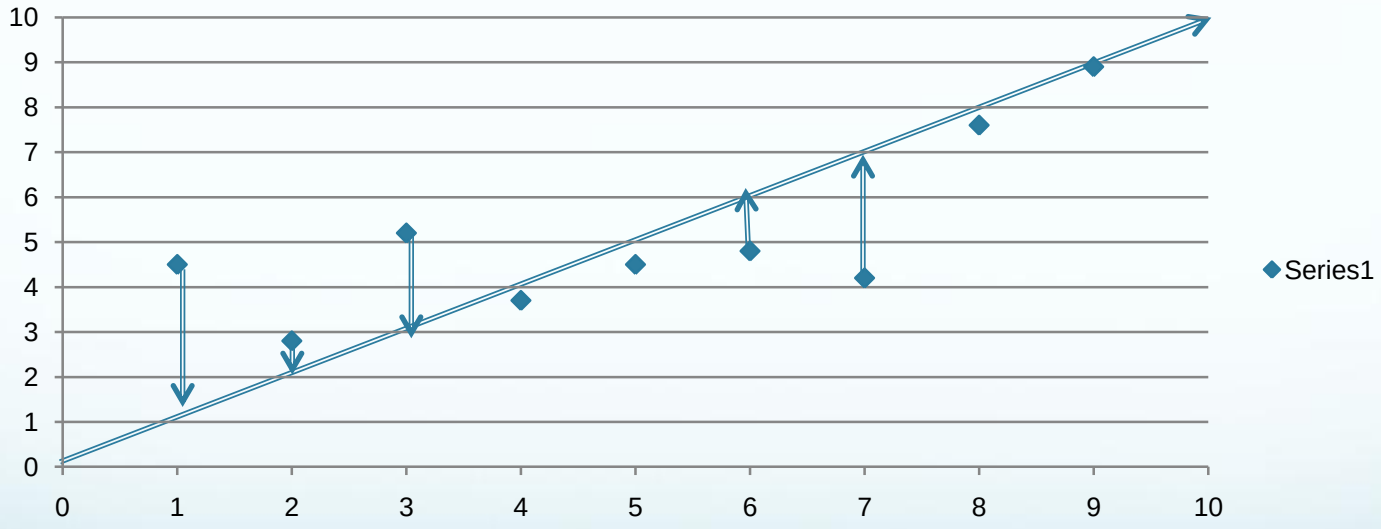
- The method of fitting a curve or line to data points so as to minimize the sum of the squares of the distances of the points from the curve or line.

- Two categories
  - Linear
  - Nonlinear

Applications

- Astronomy
- Geodesy
- Exploration

Minimize the Distance



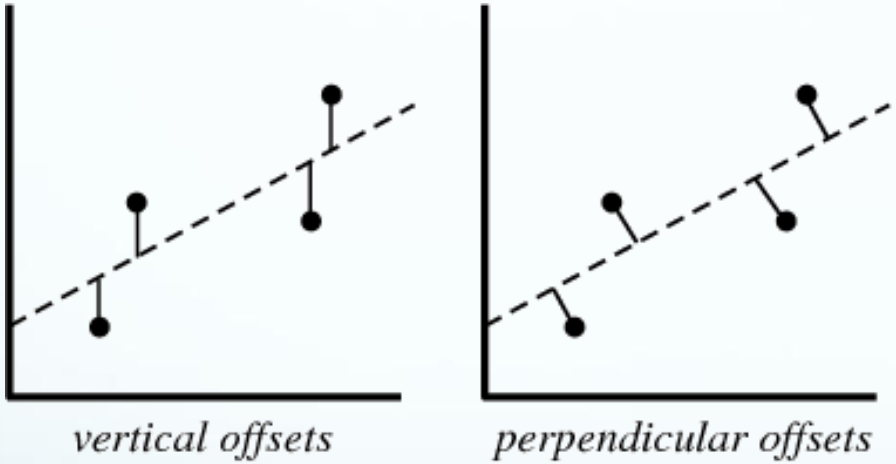
History of Least Squares

- Adrien-Marie Legendre
  - Was the first one to do so
- Carl Friedrich Gauss
  - Developed his method in 1794
  - Strength of Gauss method was proven when predicting the asteroid Ceres
  - Published his method in 1809

Ceres

- On January 1, 1801
  - Giuseppe Piazzi
  - Tracked for 40 days
  - Vanished
  - wanted to predict Ceres' reappearance without using Kepler's nonlinear equation of planetary motion
  - Gauss used his new method to predict its reappearance
  - Based on Gauss method Franz Xaver von Zach was able to relocate Ceres

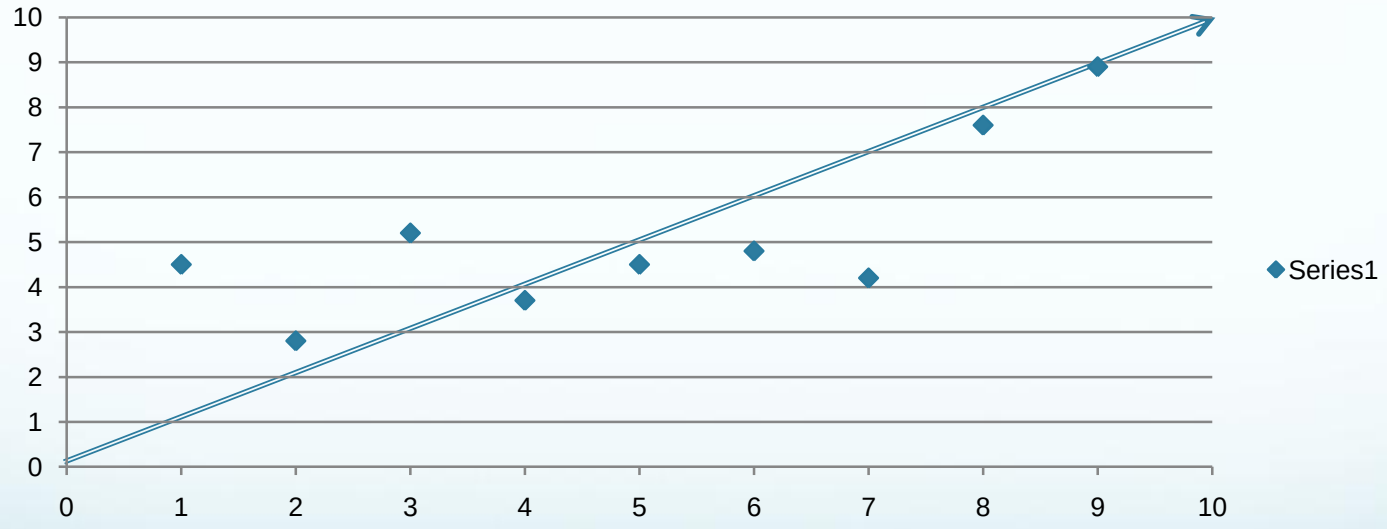
When Using Least Squares



- We use a form least squares without even knowing it.
- Always use vertical lines

How it Works

- Our brain identifies the points and drawls a best fit line that encompasses most if not all of the points



The Formula

A regression model is a linear one when the model comprises a linear combination of the parameters, i.e.

$$f(x_i, \beta) = \sum_{j=1}^{m} \beta_j \phi_j(x_i)$$

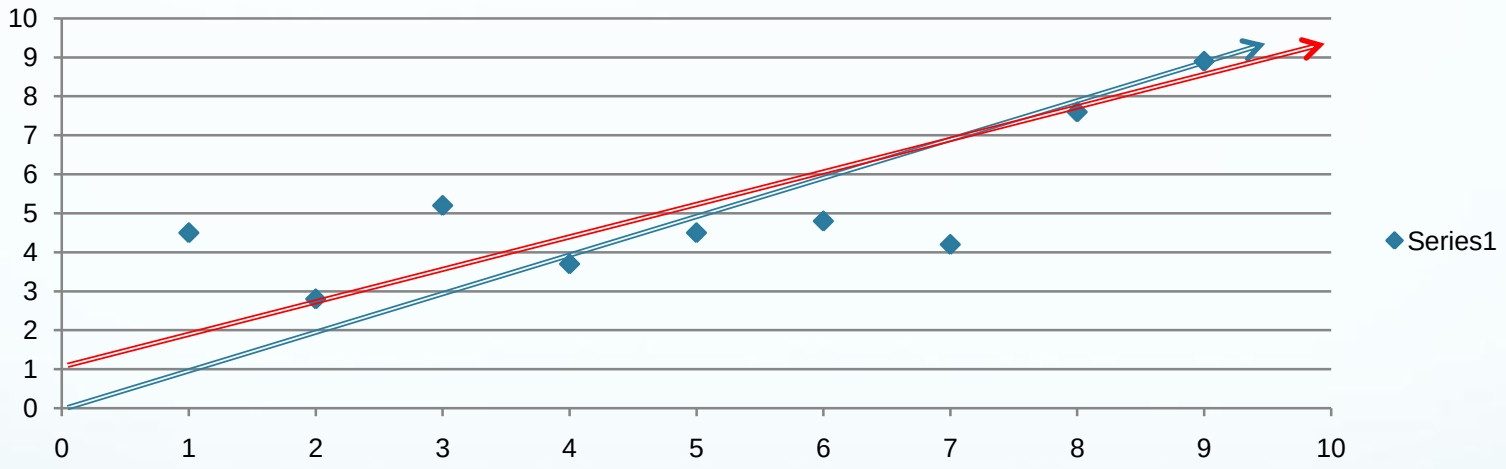
where the coefficients, φ*j, are functions of xi.Letting*

$$X_{ij} = \frac{\partial f(x_i, \boldsymbol{\beta})}{\partial \beta_j} = \phi_j(x_i).$$

we can then see that in that case the least square estimate (or estimator, in the contex of a random sample), is given by

$$\hat{\boldsymbol{\beta}} = (X^T X)^{-1} X^T \boldsymbol{y}$$

The Graph



Brain

Formula

Recap

- Mathematical method to draw a best fit line
- Developed by:
  - Adrien-Marie Legendre
  - Carl Friedrich Gauss
  - 1809
  - Used to predict the reoccurrence of an asteroid

Resources

- http://www.mathworks.com/moler/leastsquares.pdf
- http://mathworld.wolfram.com/LeastSquaresFitting.html
- http://www.stat.ucla.edu/history/legendre.pdf

Questions